4 Multilevel models

4.1 Introduction

There has been a growing interest in recent years in fitting models to data collected from longitudinal surveys that use complex sample designs. This interest reflects expansion in requirements by policy makers and researchers for in-depth studies of social processes over time.

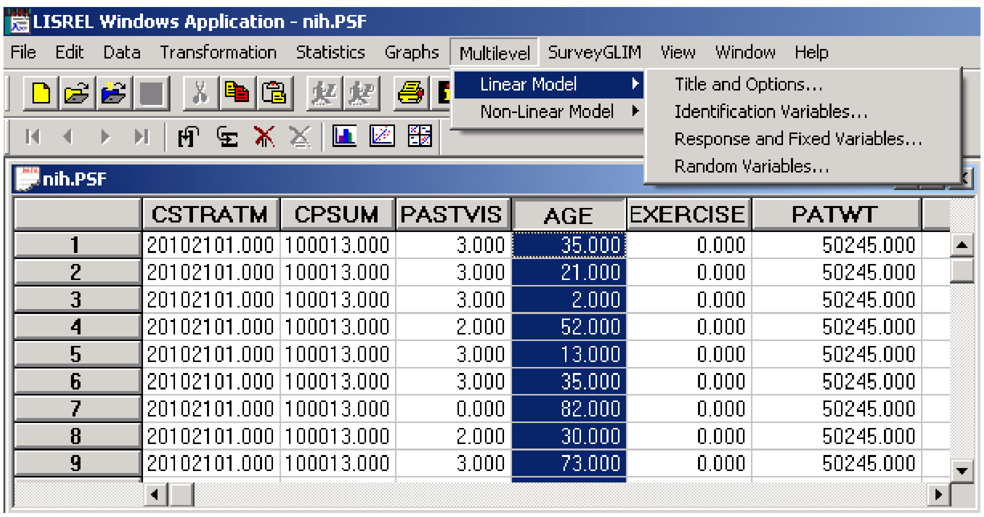
Although structural equation modeling allows for a tremendous flexibility in modeling error structures, it is in general not straightforward to analyze nested data structures with it. This, on the other hand, is a strong point of multilevel modeling, which is also more flexible than structural equation modeling when repeated measurement occasions vary between individuals. In order to address concerns regarding the appropriate analyses of survey data, the LISREL 8.7 (Jöreskog & Sörbom 2004) multilevel module features an option for users to include design weights on levels 1, 2 or 3 of the hierarchy. Correct parameter estimates and robust standard error estimates, using a Taylor linearization approach, are produced.

In this chapter, we describe and illustrate the method used to allow for weights on levels 1, 2 or 3 of the hierarchy in the multilevel module of LISREL 8.7. Section 2 is a brief overview of the graphical user interface (GUI) for the linear multilevel modeling module implemented in LISREL 8.7. Section 3 gives the multilevel syntax that is generated via the dialog boxes. For advanced users, there are additional syntax specifications presently not available via the interface dialog boxes. Practical application of a 3-level model with design weights on levels 2 and 3 of the hierarchy is given in Section 4. In Section 5 we provide evaluation and simulation studies. Section 6 describes the general weighting strategy of Pfeffermann et al. (1997), followed by a more rigorous theoretical treatment.

4.2 Graphical User Interface

4.2.1 The Multilevel Models menu

The **Multilevel Models** menu provides you access to a sequence of four dialog boxes that can be used to create a basic syntax file interactively. It is located on the PSF (PRELIS System File) window of LISREL which is used to display, manipulate and process raw data. In other words, this menu is only available when a PRELIS data file (*.**psf**) is opened. To illustrate this, the PSF window for the file **NIH1.psf** is shown below with the **Multilevel** menu expanded.

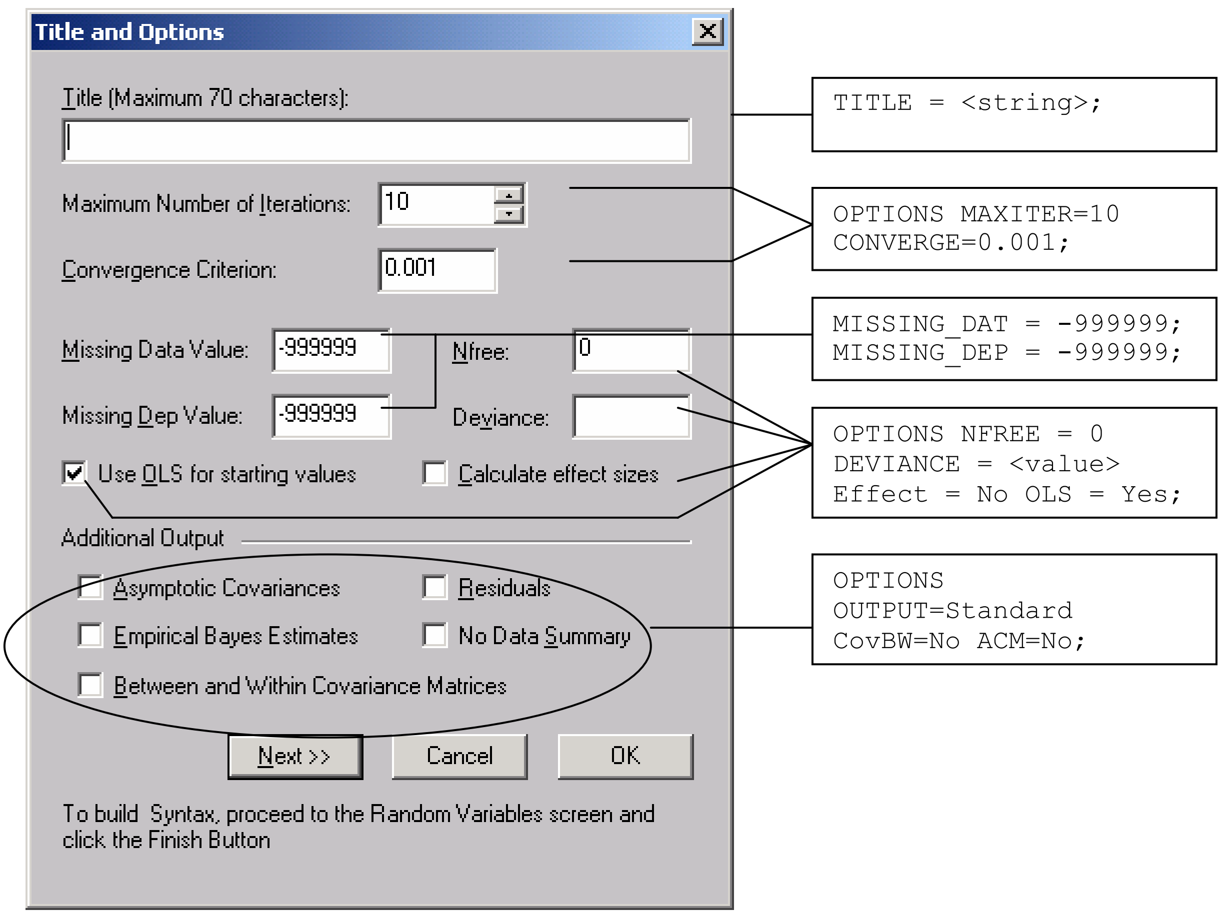


Presently, the **Multilevel Models** menu has four options that can be used to perform basic multilevel analyses. Advanced options that enable the user to specify more complex models must be typed in once an syntax file has been generated. See Section 4.3 of this guide for a description of these options.

The typical first step for using the **Multilevel Models** menu would be to click on the **Title and Option**s option to activate that dialog box (see Section 4.2.2). However, you can click on other options to go directly to the **Identification Variables** (see Section 4.2.3), **Response and Fixed Variables** (see Section 4.2.4) or **Random Variables** dialog box (see Section 4.2.5).

4.2.2 The Title and Options dialog box

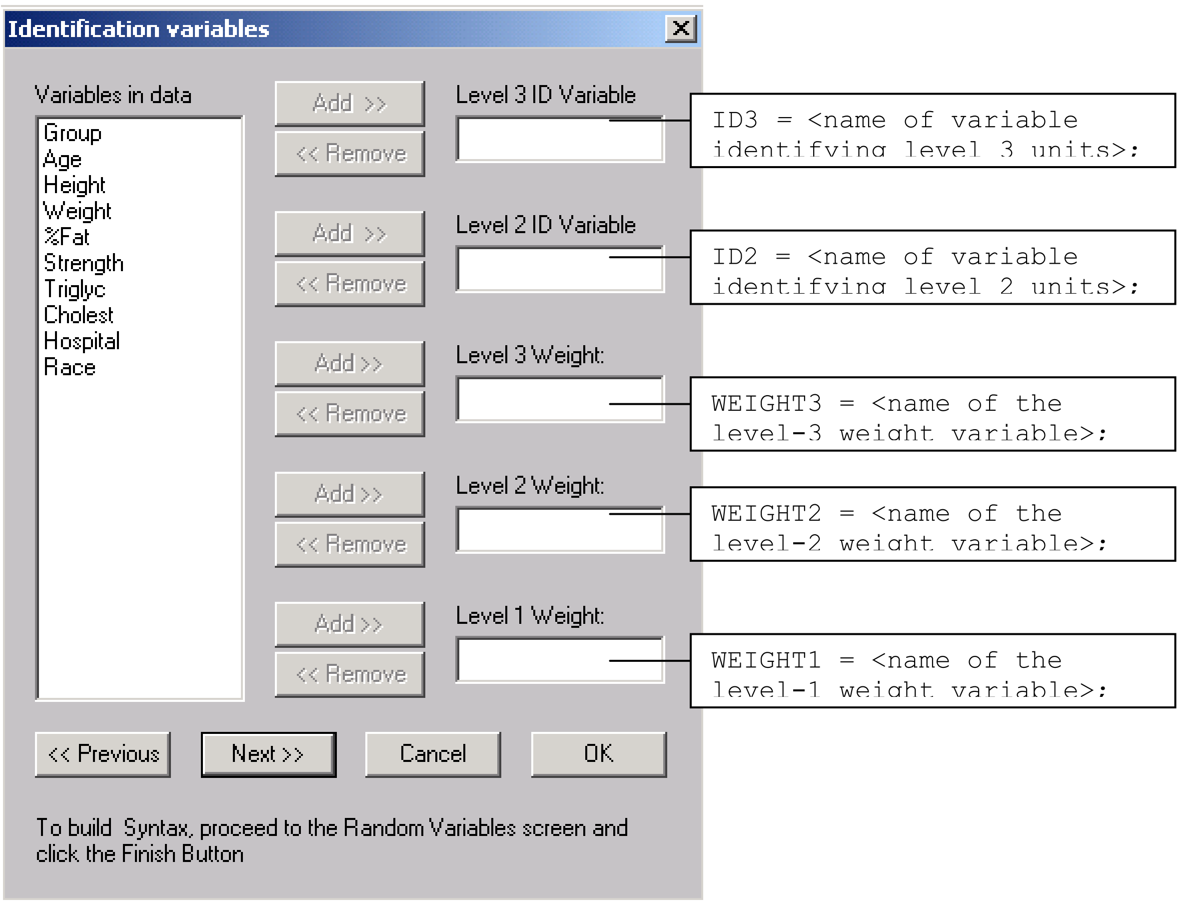
The **Title and Options** dialog box is used to provide a title for the analysis and keywords concerning the iterative procedure. The image below shows the default settings for this dialog box and, to the right, the corresponding syntax commands. See the alphabetical list of syntax commands for details on the options available other than the defaults settings: TITLE command (section 4.3.16); OPTIONS command, including the MAXITER, NFREE and OUTPUT keywords (section 4.3.12); and MISSING_DAT (section 4.3.10).



The **Next** button takes you to the **Identification of Variables** dialog box.

4.2.3 The Identification Variables dialog box

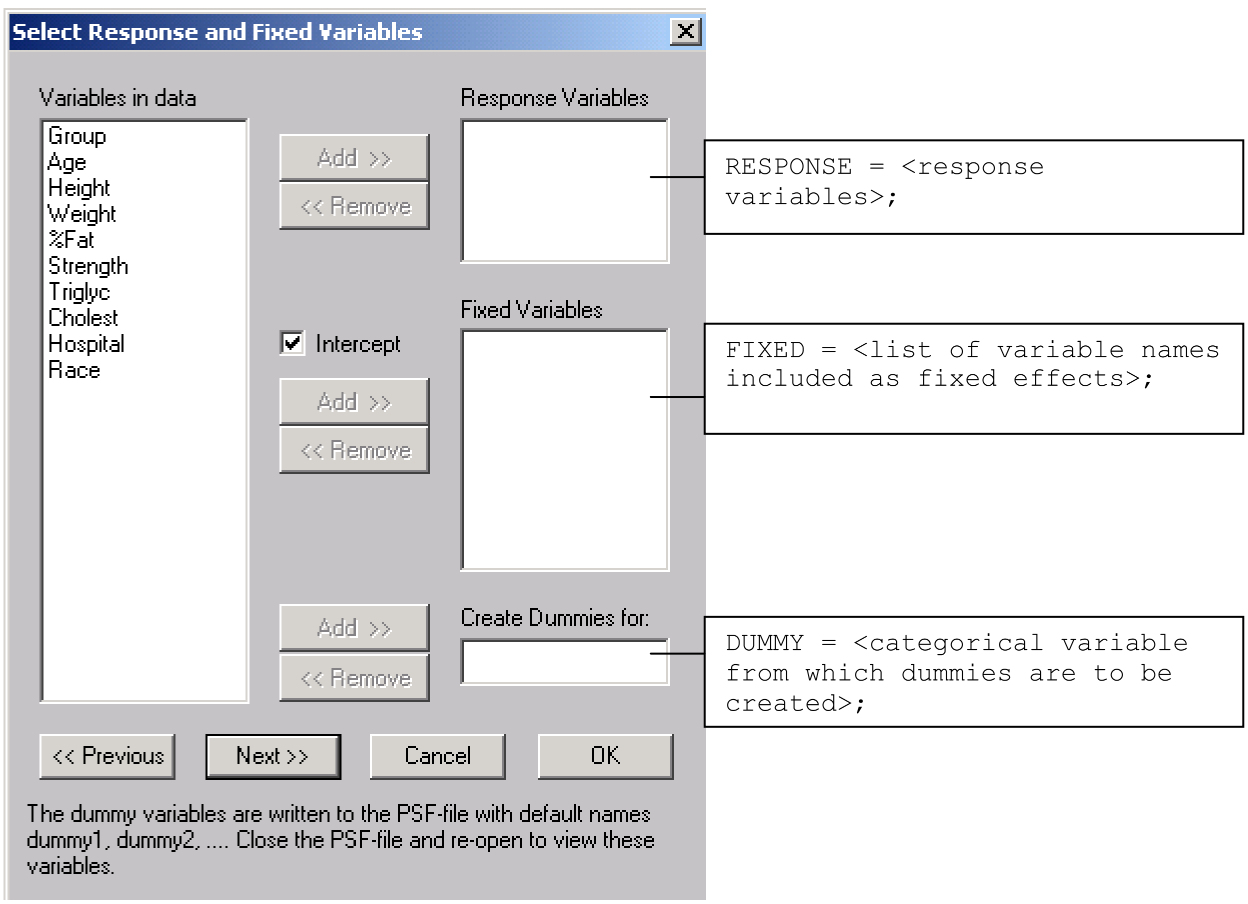
The **Identification of Variables** dialog box is used to select the variables in the PRELIS data file (*.**psf**) that identify the various levels of the hierarchy. The image below shows the default settings for this dialog box and, to the right, the corresponding syntax commands. See the alphabetical list of syntax commands for details on the options available other than the default settings: IDn command (section 4.3.9) and WEIGHTn command (section 4.3.17).



The **Next** button provides access to the **Select Response and Fixed Variables** dialog box.

4.2.4 The Response and Fixed Variables dialog box

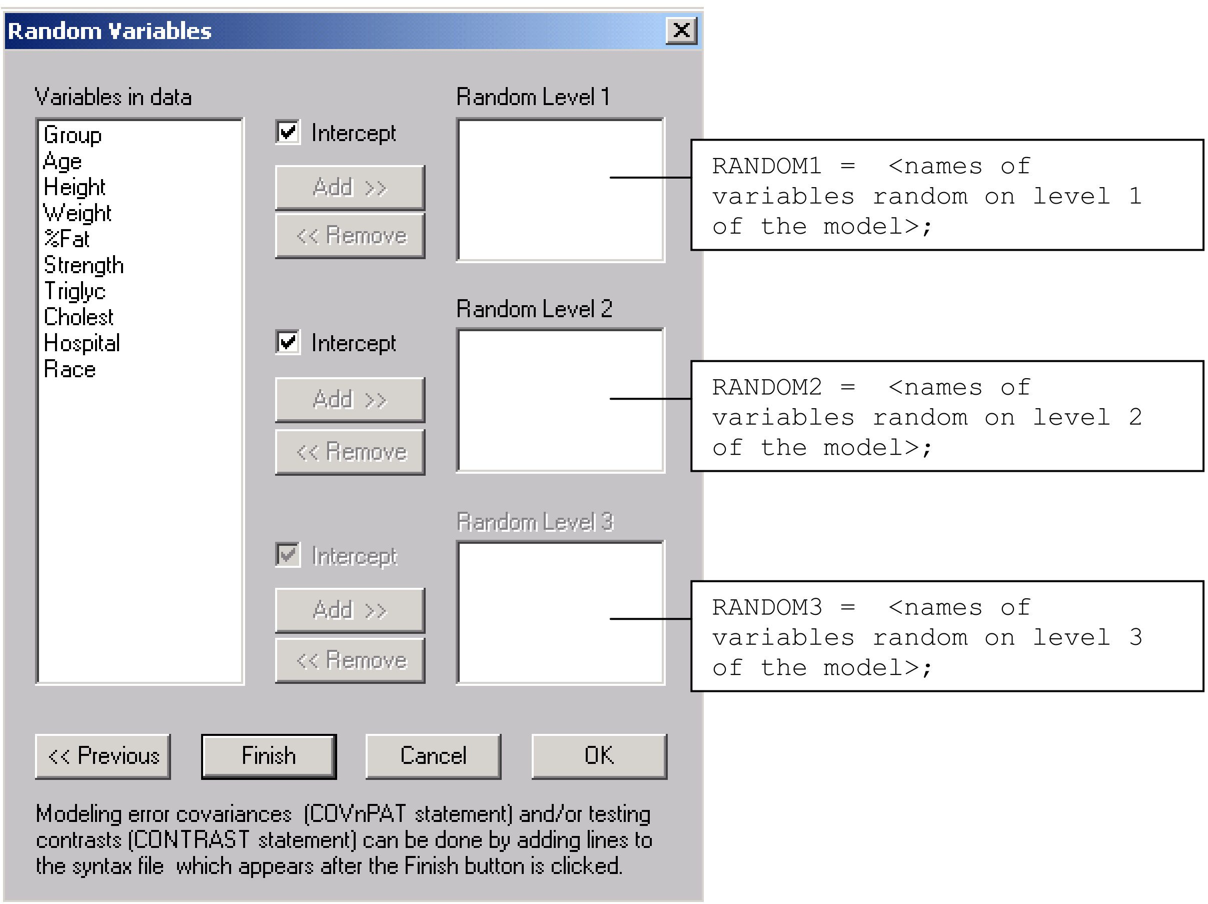
The **Select Response and Fixed Variables** dialog box is used to select the outcome and fixed variables to be included in the model from the PRELIS data file (*.**psf**). The image below shows the default settings for this dialog box and, to the right, the corresponding syntax commands. See the alphabetical list of syntax commands for details on the option available other than the default settings: RESPONSE command (section 4.3.14), FIXED command (section 4.2.6) and DUMMY command (section 4.3.5).



The **Next** button take you to the **Random Variables** dialog box.

4.2.5 The Random Variables dialog box

The **Random Variables** dialog box is used to select the variables for which coefficients are assumed to be random from the PRELIS data file (*.**psf**). Default settings for this dialog box are shown in the image below. To the right, the corresponding syntax commands are given. See the alphabetical list of syntax commands for details on the options available other than the default settings: RANDOMn command (section 4.3.13).



Once all the options are set as desired, click the **Finish** button to generate the syntax.

4.3 Syntax

4.3.1 The structure of the syntax file

The basic structure of the syntax file is as given below, and the **required** commands are indicated. In this section, commands appear in the order in which they are used in the syntax file. In the sections to follow, the commands are listed in alphabetical order.

OPTIONS;	Required
SY = name of PRELIS system file;	Required
IDn = name of variable identifying level n units;	Required
WEIGHT = label;	Optional
MISSING_DAT = real value;	Optional
MISSING_DEP = real value;	Optional
RESPONSE = name(s) of response variables(s);	Required
FIXED = names of variables included as fixed effects in the model ;	Required
RANDOMn = names of variables included as random effects on level n of the model ;	Required
TITLE = job title;	Optional
CONTRAST = name of contrast file;	Optional
COVnVAL = starting values for level n random coefficient covariance matrix;	Optional
COVnPAT = pattern for level n random coefficient covariance matrix;	Optional
FIXVAL = starting values for fixed effect parameters;	Optional
FIXPAT = pattern for fixed effect parameters ;	Optional
DUMMY = categorical variable from which dummies are to be created;	Optional

Guidelines for constructing or changing the syntax file:

When syntax is generated through the interface, the commands generated and saved to a ***.pr2** file will automatically conform to the syntax rules given in the next section. When the syntax file is constructed or edited outside the interface, the following guidelines should be kept in mind:

- All commands start with a keyword and conclude with a semi-colon.
- There is no specific required order in which commands have to be given, with the exception of the OPTIONS command, which must always be the first line in the syntax file.
- Lines may be left blank between commands.
- Multilevel commands and keywords are not case-sensitive, but variable names are.
- Not all of the available commands have to be included in the syntax file.

In LISREL 8.7, the CONTRAST, COVnPAT, COVnVAL, FIXVAL and FIXPAT commands are not available via the graphical user interface. These commands are typically used in more advanced applications and can be added by editing the syntax file, or by writing an syntax file in a text editor. The separate commands are discussed, in alphabetical order, in the next 16 sections.

4.3.2 CONTRAST command

The CONTRAST command is used to specify the path to and name of the syntax file containing information on any contrast(s) between the fixed effects in the model to be tested. This is an **optional** command.

Syntax

```
CONTRAST = <filename>;
```

where <filename> denotes the complete name (including drive and folder names) of the file containing information on the fixed effects contrasts to be tested.

Examples:

1. Specifying the filename:

   ```
   CONTRAST= C:\MLEVEL\EXAMPLES\MLEVEL.CTR;
   ```

   The drive and folder names may be omitted if the contrast file and the syntax file are in the same folder.

2. Specifying the contrasts between fixed effects in a ***.ctr** file:

   Suppose that there are six fixed effects in a particular model, these being INTERCEPT, GENDER, MATHS, READING, SCIENCE and WRITING.

   If, for example, one wishes to test

$$\begin{aligned} H_0 : \beta_{READING} - \beta_{WRITING} &= 0; \\ \beta_{MATHS} - \beta_{SCIENCE} &= 0; \end{aligned}$$

   this can be tested by specifying

$$H_0 : \mathbf{C\beta} = 0$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

and

$$\beta = \left[\beta_{INTERCEPT} \quad \beta_{GENDER} \quad \beta_{MATHS} \quad \beta_{READING} \quad \beta_{SCIENCE} \quad \beta_{WRITING} \right]$$

Note that each row of **C** has six elements, corresponding to the six fixed effects. The first contrast between fixed effects is in the first row. Since the fourth element in the first equals 1, and the sixth element is -1, this denotes a contrast between the $\beta_{READING}$ and $\beta_{WRITING}$ effects.

The contrast file will have the following form:

```
2
0  0  0  1  0 -1
0  0  1  0 -1  0
```

The first row indicates the number of contrasts and the second and third rows the actual contrasts to be tested.

If the contrast file is specified as

```
1
0  0  0  1  0  -1
1
0  0  0  0 -1   0,
```

two separate contrast tests are performed as opposed to a simultaneous test for two contrasts.

4.3.3 COVnPAT command

The COVnPAT commands are used to place constraints on the covariance matrices of random coefficients on the different levels of the model. We denote these covariance matrices by $\Phi_{(1)}$, $\Phi_{(2)}$, and $\Phi_{(3)}$ or, in general, by $\Phi_{(n)}$, $n = 1, 2, 3$.

One COVnPAT command is allowed for each level of the hierarchy. If, for instance, a 3-level model with random components on all three levels of the hierarchy is to be fitted, up to three COVnPAT commands may be included in the syntax file.

Note that on level 1, only structures pertaining to the diagonal elements of the level-1 random effects covariance matrix are permissible. The use of COVnPAT commands is **optional**.

Syntax

```
COVnPAT= <keywords>;
```

Valid keywords are as follows:

DIAG — In this case the covariance matrix of random parameters on level *n* of the model will be constrained to be a diagonal matrix.

TOEPLITZ — The covariance matrix on levels 2 or 3 will be constrained to be of the form of a so-called Toeplitz matrix, that is

$$\mathbf{\Phi}_{(n)} = \begin{bmatrix} \gamma_0 & \gamma_1 & \gamma_2 & & \\ \gamma_1 & \gamma_0 & \gamma_1 & \ddots & \\ \gamma_2 & \gamma_1 & \gamma_0 & \ddots & \gamma_2 \\ & \ddots & \ddots & \ddots & \gamma_1 \\ & & \gamma_2 & \gamma_1 & \gamma_0 \end{bmatrix}$$

INTRA — The covariance matrix of random parameters on levels 2 or 3 will be constrained to have an intra-class structure, that is

$$\mathbf{\Phi}_{(n)} = \begin{bmatrix} \alpha & \beta & \dots & \dots & \beta \\ \beta & \alpha & \beta & \dots & \vdots \\ \vdots & \beta & \alpha & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \beta \\ \beta & \dots & \dots & \beta & \alpha \end{bmatrix}$$

MA1 — Constrains the covariance matrix on level *n* to be similar to that of a time series process of order MA1. The form of the covariance matrix will then be

$$\mathbf{\Phi}_{(n)} = \begin{bmatrix} \gamma & \beta & 0 & \dots & 0 \\ \beta & \gamma & \beta & \ddots & \vdots \\ 0 & \beta & \gamma & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \beta \\ 0 & \dots & 0 & \beta & \gamma \end{bmatrix}$$

The following conventions apply to the use of the COVnPAT command:

- Any line of input may not exceed 127 characters. Thus, if a large COVnPAT matrix is entered, care should be taken that no row of this matrix exceeds this limit. If a row of the matrix is too long, it may simply be continued on the next line of the syntax file.

- If elements of the covariance matrix to be estimated are constrained to be equal in value, the number assigned to those elements must be the same.
- As with all other commands in the syntax file, the command should end with a semi-colon that may be placed directly after the last element of the matrix as specified or on the next line of the syntax file.
- The matrix specified must have the same number of elements as implied by the RANDOMn command. That is, if there are p variables listed in the RANDOMn command, a total number of $\frac{1}{2}\ p\ (p + 1)$ elements must be entered.
- In order to assign initial values to elements of the covariance matrix on level-*n* or to set fixed elements of the matrix to user specified values, the COVnPAT command must be used in conjunction with the COVnVAL command.

User-specified values

To constrain the elements of the covariance matrix to be of a form other than those discussed above, you may specify this required structure with the COVnPAT command. This can be done by entering a lower-triangular matrix with the required structure on the COVnPAT command. If, for example, the covariance matrix corresponding to the RANDOMn command

```
RANDOMn = X1 X2 X3 X4;
```

is to be constrained, it can be accomplished by following a row-wise numbering convention for the lower triangular elements of the covariance matrix as shown below.

```
1
2  3
4  5  6
7  8  9  10
```

The elements to be fixed are then replaced with "0". If, for example, the matrix is constrained to be diagonal, the command to be used is as follows:

```
COVnPAT = 1
          0  3
          0  0  6
          0  0  0  10;
```

The structure as specified indicates that there are four parameters to be estimated (*i.e.* numbers 1, 3, 6 and 10, corresponding to the variances) and six fixed parameters (corresponding to the covariances), indicated by 0. The values which the fixed parameters are to be set equal to can be supplied using the COVnVAL command. If the COVnVAL command is omitted, the fixed parameters will be constrained to be equal to zero, as the initial structure of all covariance matrices are assumed to be diagonal at the start of the iterative procedure.

Examples:

1. In the case of an MA1 process, for example, the command will be as follows:

```
COVnPAT = 1
          2  1
          0  2  1
          0  0  2  1;
```

From this structure it follows that there are only two parameters to be estimated (numbers 1 and 2) while all other parameters are constrained to be equal to zero, unless otherwise specified using the COVnVAL command.

2. It is permissible to constrain diagonal elements of the level-n covariance matrix to be fixed through the use of the COVnVAL command.

The following commands, for example, are permissible:

```
COVnPAT = 1
          2  0
          4  2  0
          0  0  2  0;
COVnPAT = 0
          2  0
          4  2  0
          0  0  2  0;
```

Note that 0-values indicate that the corresponding elements remain fixed at the values specified in the COVnVAL paragraphs.

4.3.4 COVnVAL command

COVnVAL commands may be used to provide either initial values for elements of the covariance matrix on level *n* of the model or to provide values for elements fixed through the use of keywords of the COVnPAT command. Note that the use of COVnVAL commands is **optional**.

One COVnVAL command is allowed for each level of the hierarchy. If, for instance, a 3-level model with random coefficients on all three levels of the hierarchy is to be fitted, up to three COVnVAL commands may be included in the syntax file.

The values to be used for the elements of the covariance matrix must be entered in the form of a lower-triangular matrix. The number of values entered must be the same as the number of elements implied by the relevant RANDOMn command. If there are p variables listed in the RANDOMn command, ½ p (p + 1) values must be entered. If a large number of values is entered, a row of the lower-triangular matrix may be continued on the next line of the syntax file if the number of characters in that row of the matrix exceeds 127 characters. The command must end with a semi-colon, which may be entered on the last line of the values given or on the next line of the syntax file.

Syntax

```
COVnVAL = <values specified by user>;
```

Examples:

1. Providing values for the elements of the covariance matrix:

   Continuing with the example used to illustrate the use of the COVnPAT command to obtain a user specified covariance structure, the following command illustrates how you may provide values for the elements of the covariance matrix (n):

```
COVnVAL = 1.00
          0.32  0.85
          0.63  0.62  0.78
          0.19  0.00  0.25  0.99;
```

   If an accompanying COVnPAT command is not used, these values will function as starting values for the level-n covariance matrix. When good starting values for the elements of this covariance matrix are known, the use of the command as shown above together with the use of the keyword OLS = NO in the OPTIONS command could decrease the number of iterations required to obtain convergence.

2. Specifying a diagonal structure for a covariance matrix:

   When the command

```
COVnPAT = DIAG;
```

   is used together with the COVnVAL command given in the previous example, the values specified on the diagonal of the lower-triangular matrix will be used as initial values for the parameters which are to be estimated. The off-diagonal elements of the covariance matrix will then be constrained to be equal to the values of off-diagonal elements of the matrix given above.

4.3.5 DUMMY command

The DUMMY command is used to create dummy variables for a selected variable. Names for the dummy variables are denoted by dummy1, dummy2, …, dummyk, where k equals the number of distinct values of the selected variable. Use of the DUMMY command is **optional**.

Syntax

```
DUMMY = <varname>;
```

Example:

```
DUMMY = TIME;
```

Note:

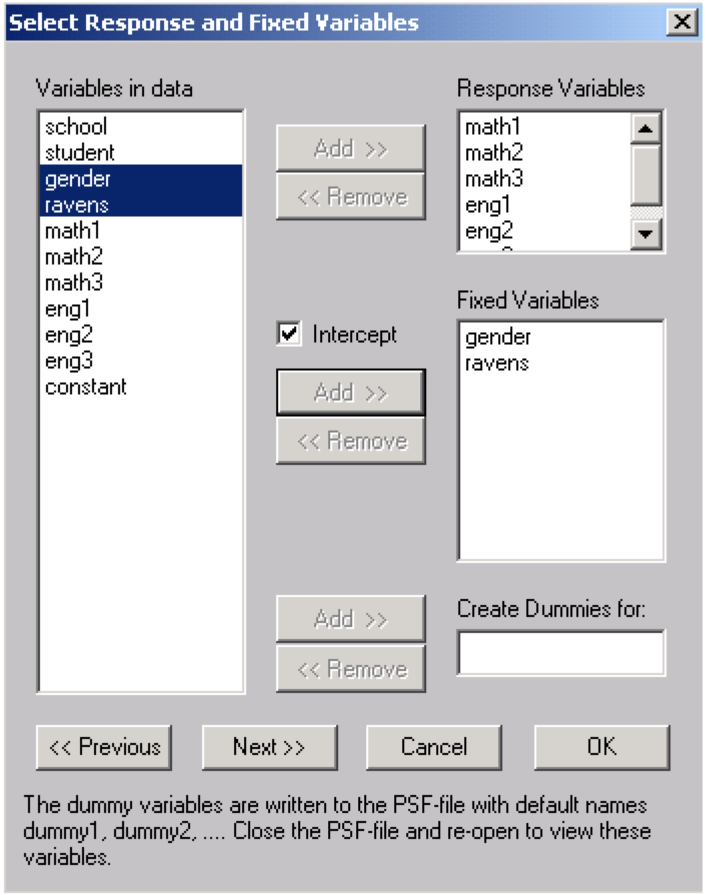
- If the variable TIME has 4 distinct values, 0, 1, 2, and 3, then the command above will result in the creation of four dummy variables: dummy1, dummy2, dummy3, and dummy4.
- You can change the default names of the dummy variables by the inclusion of the PREFIX keyword on the DUMMY command. For example:

```
DUMMY = TIME PREFIX = TIM;
```

- In this case, the dummy variable names will be TIM1, TIM2, TIM3, and TIM4.

4.3.6 FIXED command

The FIXED command is used to identify the fixed effects for the model to be analyzed. When the syntax file is created through the interface, the FIXED command is automatically generated. If, however, the file is edited manually, the guidelines below should be followed. Identification of the fixed effects are done on the **Select Response and Fixed Variables** dialog box (see below). The FIXED command is a **required** command, and must appear in any syntax file.



Syntax

FIXED = <list of variables names to be included as fixed effects>;

The fixed effects may be all of the predictor variables contained in the raw data file or any subset of these predictors and may be specified in any order. Variable names are case sensitive and thus spelling of the names must correspond to the spelling used in the data spreadsheet (*.**psf** file).

If a covariate is included in the analysis, this should be reflected in the FIXED command. The format in which the covariate should be entered is as follows:

FIXED = intcept1 var1 var2 . . . varn covariate covariate*var1 covariate*var2 . . . covariate*varn;

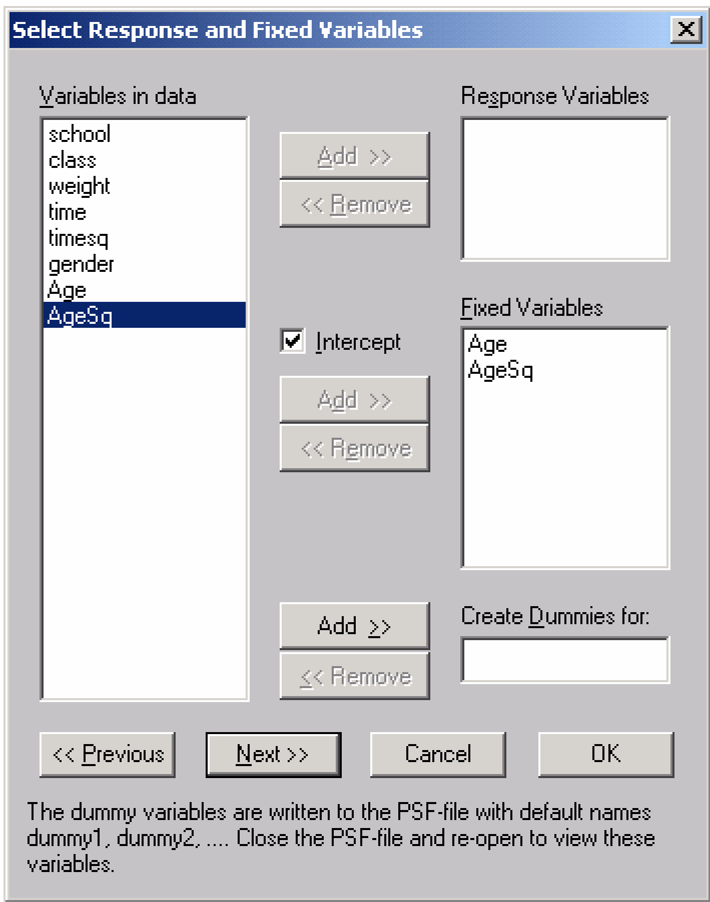
The covariate can be used in combination with any of the predictors listed in the FIXED paragraph. Note, however, that the multilevel module accepts a FIXED command of the form FIXED = var1*var2; The specification var1*var2 cannot be generated by the interface, only by manually editing the syntax file. A specification of the form var1*var2*var3 is not presently allowed.

Initial estimates for the fixed effects may be provided manually. This is done through use of the **optional** FIXVAL command that will be discussed in Section 4.3.8. See also Section 4.3.7 for a description of the FIXPAT command.

Examples:

FIXED = INTCEPT AGE AGESQ;
FIXED = Dummy1:Dummy6;

or any other similar command. The first command shown corresponds to the settings in the **Select Response and Fixed Variables** dialog box shown below.



If the variable GENDER is to be included as covariate in (1), the appropriate FIXED command is as follows:

FIXED = intcept AGE AGESQ GENDER GENDER*AGE GENDER*AGESQ;

4.3.7 FIXPAT command

To specify a patterned structure for the vector of fixed parameters, the FIXPAT command may be used, with or without an additional FIXVAL command (see Section 4.3.8). Use of this command is **optional**.

Syntax

```
FIXPAT = <list of numbers>;
```

where <list of numbers> denotes a list of positive integers separated by blank spaces. The number of values entered must equal the number of predictors in the model.

Examples:

1. Constraining fixed effects to be equal:

```
FIXPAT = 1 1 3 3 5 6;
```

This statement specifies that the vector of six parameters in the fixed part of the model are constrained as follows: BETA1 = BETA2; BETA3 = BETA4 while BETA5 and BETA6 are estimated freely. In the command shown above, the actual numbers correspond to the order of the parameter in question: "1" denotes the first parameter, "3" the third and "5" and "6" the fifth and sixth of the parameters in the fixed part of the model.

2. Fixing fixed effects to user-specified values:

```
FIXPAT = 0 0 3;
```

If '0' values are in the list of numbers, then the FIXPAT command should be used in conjunction with the FIXVAL command. If, for example, FIXVAL = 10 2.5 0.15; then BETA1 and BETA2 are fixed at their initial values (10 and 2.5 respectively) while BETA3 is estimated freely.

4.3.8 FIXVAL command

It is also possible to provide initial values for the fixed parameters in the model to be analyzed. This may be achieved with the FIXVAL command, which allows you to provide starting values for these parameters. The use of the FIXVAL command and the OLS = NO keyword of the OPTIONS command may be particularly effective when good starting values of these parameters are available. Use of this command is **optional**.

Syntax

```
FIXVAL = <as specified by user>;
```

The number of values entered by you using this command must be equal to the number of fixed parameters to be estimated. There is no specific format in which the values have to be entered.

Example

The commands

```
FIXVAL = 0.151 0.355 0.654;
FIXVAL = 0.151
         0.355
         0.654;
```

and

```
FIXVAL = 0.151  0.355
         0.654
         ;
```

are all permissible. If the first of these commands is used and the number of characters in the user specified string exceeds 127 characters, the next line of the syntax file should be used.

4.3.9 IDn command

The ID command(s) are used to indicate the variable(s) identifying the units on the different levels of the hierarchy. ID command(s) are **required** command(s).

If the model specified is a 2-level model, the command ID2 is required. Likewise, if a level-3 model is to be considered, the ID2 and ID3 commands are required in the syntax file.

Variables listed in the ID commands must be included in the data file (*.**psf** file). Variable names are case sensitive, therefore the spelling and case in which they are given need to correspond to that given in the spreadsheet.

Syntax

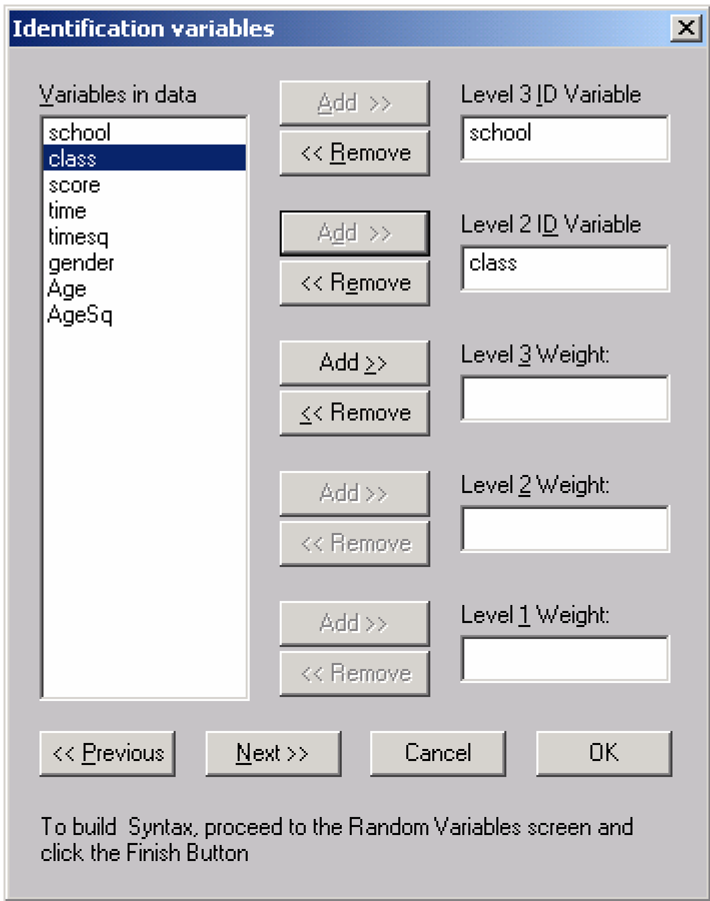
```
IDn = <variable name identifying level-n units>;
```

Example

Suppose the raw data file contains information on the test scores, age and gender of pupils belonging to classes within schools, and the variables school, class, pupil, age, gender and score are contained in the data file. The following ID commands may be used to identify the levels of the hierarchical structure:

```
ID3 = school;
ID2 = class;
```

The **Identification Variables** dialog box shown below shows the settings needed to obtain these commands.



4.3.10 MISSING_DAT command

The MISSING_DAT and MISSING_DEP commands may be used when missing data are present in the raw data file. The MISSING_DAT command allows you to specify a numeric value, which will represent a missing value on any of the variables used in the analysis. This command may also be used in conjunction with the MISSING_DEP command, as described in Section 4.3.11. Note that use of the MISSING_DAT command is **optional**.

Syntax

```
MISSING_DAT = <value>;
```

Any positive or negative value may be used. Only one value is allowed in this command. All records with data values equal to the code specified in this command will subsequently be removed from the analysis.

Default

-999999.0.

Examples

Valid examples of the use of this command include the following:

```
MISSING_DAT = 99;
MISSING_DAT = -998.0;
MISSING_DAT = 0;
```

4.3.11 MISSING_DEP command

The MISSING_DEP command may be used to specify a code assigned to missing values on the response variables only. The consequence of using the MISSING_DEP command is that only records with response variable values equal to the code assigned through the MISSING_DEP command will be removed from the analysis. Note that use of this command is **optional**.

The MISSING_DEP command is recommended for use in the case of multivariate analysis. If only one of the response variables to be used in the multivariate analysis has a missing response, only that particular response will be considered missing while the remaining responses will still be used.

Syntax

```
MISSING_DEP = <value>;
```

Any positive or negative value may be used. Only one value is allowed in this command. All records with response variable values equal to the code specified in this command will subsequently be removed from the analysis.

Default

-999999.0.

Example

Consider the observations

Response variables				Predictor variables			
4.0	5.3	1.7	99	1	10	14.5	999
3.2	4.4	99	7.7	3	12	13.7	53.2

and the command

```
MISSING_DEP = 99;
```

If the code 99 is identified as the code for missing data values on the dependent variables, this will imply that the analysis of this record will use the first three response values and disregard the fourth one in the case of the first observation. The third response variable will be omitted for the second observation.

If, additionally, the code 999 is specified (MISSING_DAT = 999) as the code for missing data values on all the variables included in the analysis, the whole first record as given above will be deleted from the data set to be analyzed. The second observation will be retained with the exception of the third response variable value.

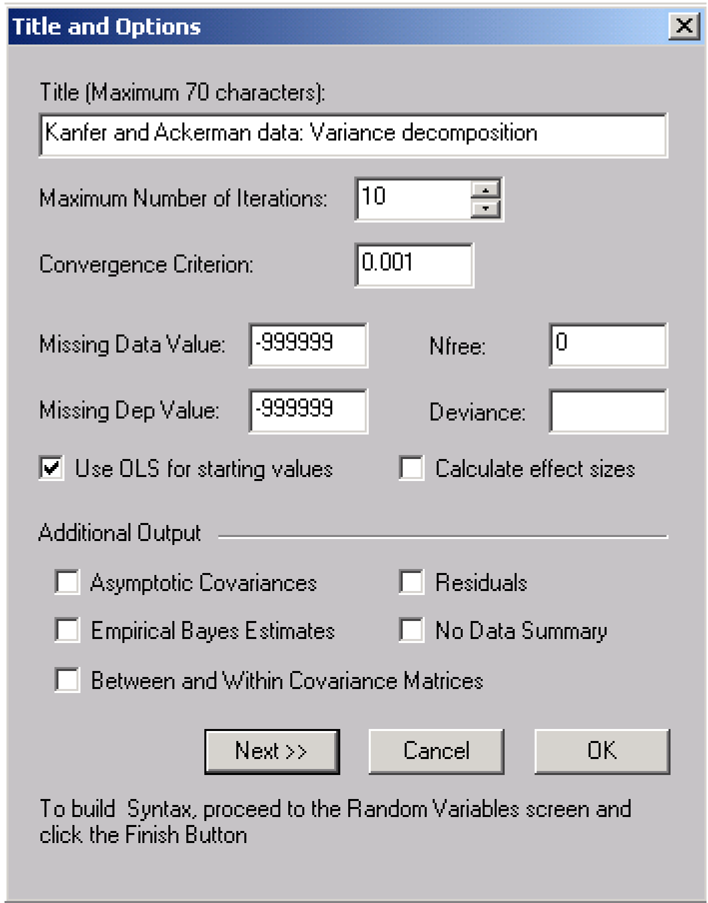
This is accomplished by using both the MISSING_DEP and MISSING_DAT commands as follows:

```
MISSING_DEP = 99;
MISSING_DAT = 999;
```

Note that if only the MISSING_DEP command is used for the two observations given above, the value of 999 for the last predictor variable on the first observation will be considered valid data and will be used as such in the analysis.

4.3.12 OPTIONS command

Each problem for a multilevel analysis starts with an OPTIONS command. The keywords of the OPTIONS command are used to control the estimation procedure and the amount of output to be written at convergence of the iterative procedure. All keywords are set via the **Title and Options** dialog box through selection of the **Title and Options** option from the **Multilevel Models** menu on the main menu bar. Inclusion of an OPTIONS command in a syntax file is **required**.



Syntax

```
OPTIONS <keywords>;
```

The keywords and options that may be used with the OPTIONS command are listed below. Details on each keyword, option, and its default value are provided below. In the generated syntax file, keywords may occur in any order. If any OPTIONS keywords are not given, the default values will be used. Also see the examples of OPTIONS commands on p. 167 of this guide.

ACM	Requests printing of asymptotic covariance matrices
Converge	Sets a value for the test for convergence made at the end of each iteration
CovBW	Requests printing of the within- and between-clusters covariance matrices
Deviance	Provides the value of $-2\ln L$ as reported in a previous analysis, with the purpose to obtain a chi-square test statistic for comparing two nested models
Effect	Estimates and print indirect effects of coefficients in the fixed part of the model
Maxiter	Indicates the maximum number of iterations to be performed

Nfree	Indicates the number of free parameters as reported in a previous analysis
OLS	Indicates whether OLS estimates are to be calculated during the first iteration
Output	Sets the amount and type of output required
Summary	Requests printing of summary table containing sample sizes of units

ACM keyword

The ACM keyword is used to print the large-sample covariance matrices of the estimated parameters in the fixed part and random part of the model. This keyword is controlled from the **Title and Options** dialog box.

Standard errors of the estimated parameters are equal to the square roots of the diagonal elements. The non-duplicated elements of these asymptotic covariance matrices are written to external files with the following default names:

<Outputfilename>_random.acm

<Outputfilename>_random.acm

If the output file name is, for example, **kanfer1.out**, then the large-sample covariance matrices are saved to the files **kanfer1_fixed.acm** and **kanfer1_random.acm**.

Syntax

```
ACM = Yes/No
```

Default

No: asymptotic covariance matrices will not be printed.

Example

In the OPTIONS command below, the ACM keyword is used to request the printing of the asymptotic covariance matrices at convergence. A convergence criterion of 0.0001 is set as the requirement for convergence, and 30 iterations is indicated as the maximum number of iterations to be performed.

```
OPTIONS MAXITER = 30 CONV = 0.0001 ACM = Yes;
```

CONVERGE = Keyword

A test for convergence is made at the end of each iteration. If the absolute difference between the estimated parameters and their previous values are all smaller than the convergence criterion, convergence is said to have been reached. In the **Title and Options** dialog box shown above, the default value is automatically shown next to **Convergence Criterion**. To change the value, click in the box and enter the required convergence criterion.

Syntax

CONVERGE = <value>

Default

0.001 (10^{-2}).

Example

In order to use a value of, for example, 0.0001 as convergence criterion, the keyword CONVERGE = 0.0001 must be included as part of the OPTIONS command, as shown in the following example:

OPTIONS MAXITER = 10 CONVERGE = 0.0001;

The iterative procedure will terminate if this requirement is met, or if 10 iterations (set with MAXITER the keyword described below) have been performed without meeting this requirement.

COVBW keyword

The COVBW keyword is used to request printing of the within-clusters and between-clusters matrices of the random effects. The non-duplicated elements of these matrices are written to external files with the following default names:

<Outputfilename> _between.cov
<Outputfilename> _within.cov

For example, **jsp1.pr2** in the **MLEVELEX** folder. For this syntax file, the default names are **jsp1_between.cov** and **jsp1_within.cov**. This keyword is applicable to multivariate response models only and is controlled from the **Title and Options** dialog box (see example above).

Syntax

COVBW = Yes/No

Default

No: the within- and between-clusters covariance matrices will not be printed.

Example

In the OPTIONS command below, the COVBW keyword is used to request the printing of the within- and between-cluster matrices at convergence, for which a convergence criterion of 0.0001 is set as the requirement for convergence, to be attained within a maximum number of 30 iterations.

```
OPTIONS MAXITER = 30 CONVERGE = 0.0001 COVBW = Yes;
```

DEVIANCE keyword

The DEVIANCE keyword is used to provide the value of – 2 log likelihood as reported in a previous analysis, in order to obtain a χ^2 test statistic for comparing two nested models. The χ^2 statistic is defined as the difference in the deviance statistics for the two models, and has as associated degrees of freedom the difference in the number of parameters estimated in the models compared. It must be accompanied by the NFREE keyword, which is used to indicate the number of parameters estimated in the previous model. The DEVIANCE and NFREE keywords are controlled from the **Title and Options** dialog box (see earlier example of a **Title and Options** dialog box).

Syntax

```
DEVIANCE = <value>
```

where value equals the deviance (- 2 log L) value at convergence printed to the output file of the previous analysis.

Default

None: no – 2 log likelihood value provided.

Example

In the OPTIONS command below, the DEVIANCE keyword indicates that a – 2 log likelihood value of 22735.524 was obtained in the previous analysis, and that 44 parameters were estimated (NFREE = 44). See Section 4.4.2 for a detailed example.

```
OPTIONS NFREE = 44 DEVIANCE = 22735.524;
```

EFFECTS keyword

The EFFECTS keyword is used to estimate and print indirect effects of coefficients in the fixed part of the model. This keyword is controlled from the **Title and Options** dialog box.

Syntax

EFFECTS = Yes/No

where Yes indicates that indirect effects will be computed and listed for all the predictors in the model.

Default

No: Indirect effects will not be computed.

Example

1. When the OPTIONS command shown below, with EFFECTS keyword set to Yes, is used in combination with accompanying FIXED command, indirect effects will be computed and listed for the predictors INTCEPT, AGE, and AGESQ.

   OPTIONS EFFECTS = Yes;
   FIXED = INTCEPT AGE AGESQ;

2. Typical output generated in the case of EFFECTS = Yes is shown below.

```
--------------------------------------------------------------------------
COEFFICIENTS  LEVEL  T-SQUARED     APPROX DF     EFFECT SIZE
--------------------------------------------------------------------------
ability       1       50.83568     839           0.23902
intcept       2        4.51051     138           0.17791
time          2      520.05345     138           0.88898
timesq        2      284.90950     138           0.82079

Effect sizes are obtained by replacing the Z-values reported in the fixed part of the model
by t-values with DF as listed above.

Effect Size = sqrt[tsq/(DF + tsq)]

Level = 2: DF equals the number of level2 units - the number of level2 random coefficients
- the number of level2 covariates associated with level2 random coefficients.
```

MAXITER keyword

The keyword MAXITER is used to indicate the maximum number of iterations to be performed. The value of the keyword is set on the **Title and Options** dialog box (see Section 4.3.2). To change the value, click in the box and enter the required maximum number of iterations.

Syntax

MAXITER = <value>

Default

10.

The default number of iterations should be sufficient for convergence to be reached in most cases. If, however, a more stringent convergence criterion is used or previous experience with a particular data set indicates slow convergence, this keyword may be used to increase the maximum number of iterations. If, on the other hand, you wish to obtain only the OLS estimates calculated in the first iteration, MAXITER may be set equal to 1.

Example

In the OPTIONS command below, MAXITER is set to 30, indicating that a maximum of 30 iterations should be performed. The iterative procedure may terminate before this number is reached if the convergence criterion of 0.0001 (CONVERGE = 0.0001) is met.

```
OPTIONS MAXITER = 30 CONVERGE = 0.0001;
```

NFREE keyword

The NFREE keyword is used to denote the number of free parameters as reported in a previous analysis, in order to obtain a χ^2 test statistic for comparing two nested models. The χ^2 statistic is defined as the difference in the deviance statistics for the two models, and has as associated degrees of freedom the difference in the number of parameters estimated in the models compared. It must be accompanied by the DEVIANCE keyword, which is used to provide the value of – 2 log likelihood as reported in the previous analysis. The DEVIANCE and NFREE keywords are controlled from the **Title and Options** dialog box.

Syntax

```
NFREE = <number>;
```

where number is the number of free parameters, that is, the total number of parameters estimated during the previous analysis, as reported in the output file.

Default

None: no parameters indicated for previous model.

Example

In the OPTIONS command below, the NFREE keyword indicates that 44 parameters were estimated in the previous model, with a – 2 log likelihood value of 22735.524 (DEVIANCE = 22735.524).

```
OPTIONS NFREE = 44 DEVIANCE = 22735.524;
```

See Section 4.4.2 for a detailed example.

OLS keyword

OLS estimates of the fixed effects are calculated as a first step of the iterative procedure unless otherwise specified. The OLS keyword is used to indicate whether the OLS estimates are to be calculated during the first iteration. On the **Title and Options** dialog box, the default value for this keyword is shown for **Use OLS for Starting Values**.

Syntax

```
OLS = <Yes/No>;
```

If starting values (see the FIXVAL command described in Section 4.3.8) are provided, use the OLS = NO option. To set OLS to NO, use the check box on the **Title and Options** dialog box.

Default

Yes: OLS estimates will be calculated during the first iteration.

Example

As starting values are provided for the fixed effects on the FIXVAL command, the OLS keyword is set to NO on the OPTIONS command below.

```
OPTIONS OLS = No;
FIXVAL = 0.151 0.355 0.654;
```

OUTPUT keyword

The OUTPUT keyword determines the amount of output produced. The output options are controlled from the **Title and Options** dialog box. To get more than the default output, check one or both of the boxes next to **Residuals** or **Empirical Bayes Estimates** (see description of the valid options listed below).

Syntax

OUTPUT = <option>;

where the valid options are as follows:

STANDARD	The default output only
BAYES	The default output and empirical Bayes estimates
RESIDUAL	The default output and residuals
ALL	The default output, residuals and empirical Bayes estimates.

Details on each of these options are given below.

Default output (STANDARD):

The following information is written to the default output file:

1. Input specifications as supplied by you in the syntax file.
2. A summary of the hierarchical structure of the raw data.
3. Details of the iterative procedure at iteration 1 and at convergence, or MAXITER if convergence was not attained. For each iteration, aside from the first iteration, these details include the estimates, their standard errors, *z*-values and exceedance probabilities.
4. The covariance and correlation matrices of the random parameters on the different levels of the model.
5. The value of –2 log likelihood (deviance) at each iteration and number of parameters estimated.
6. The CPU time for completion of the iterative procedure and writing of required results to the output file.

Empirical Bayes estimates (BAYES):

If OUTPUT = BAYES is specified, (1) to (6) are written to the output file. One, or in the case of a 3-level model, two additional output files are also created.

The empirical Bayes estimates on levels 2 and 3 of the model are calculated and, along with their variance and relevant variable codes, are written to the files *.**ba2** and *.**ba3**, where these file names refer to the second and third level of the hierarchy respectively. The filename and path are the same as for the **.out** file.

Residuals (RESIDUAL):

If OUTPUT = RESIDUAL is specified, (1) to (6) are written to the output file. An additional file, ***.res**, is created, and contains the residuals as at convergence. The following information is given: observed *y*-values, predicted *y*-values, and residuals.

All output (ALL):

All of the above files are created.

Example

1. By using the OPTIONS command shown below (without keywords), the convergence criterion will be 0.001, a maximum number of 10 iterations will be carried out and partial output will be written to the output file ***.out**. OLS estimates will be calculated during the first iteration.

```
OPTIONS;
```

2. Use of the command shown below will exclude the calculation of the OLS estimates during the first iteration. The convergence criterion is 0.0001 and the maximum number of iterations is 20. Lack of convergence will be noted in the default output file. All output files (standard output, Empirical Bayes estimate and residual files) will be created based on the solution obtained at termination.

```
OPTIONS OLS=NONE MAXITER=20 OUTPUT=ALL CONVERGE=0.0001;
```

SUMMARY keyword

The SUMMARY keyword is used to suppress the printout of the data summary table. This keyword is controlled from the **Title and Options** dialog box. In the example of a **Title and Options** dialog box (see above), the **No data summary** check box is not checked, indicating that the SUMMARY keyword is not used.

Syntax

SUMMARY = Yes/No

Default

Yes: the summary table containing sample sizes of units within the various levels of the hierarchy is printed.

Example

The OPTIONS command below request use of the default values for the OLS, MAXITER, and CONVERGE keywords, along with suppression of the printing of the summary table, as indicated by the absence of the SUMMARY keyword.

```
OPTIONS OLS=YES MAXITER=10 CONVERGE=0.001;
```

4.3.13 RANDOMn command

The RANDOMn command is used to identify those variables whose coefficients are allowed to vary randomly over a given level of the hierarchy. One RANDOM command is allowed for each level of the hierarchy. When the syntax file is created through the interface, the RANDOM command(s) are automatically generated. Variables listed, except for the variable intcept (intercept), must be included in the data spreadsheet (*.**psf** file). The spelling and case in which they are given need to correspond to that given in the spreadsheet. By default, the intercept is automatically included as a random effect at each level of the model. To exclude the intercept term at any level, the corresponding **Intercept** check box (see the **Random Variables** dialog box below) must be unchecked. At least one RANDOMn command is **required** if a 2-level model is fitted. For a 3-level model, two RANDOMn commands must be included in the syntax file.

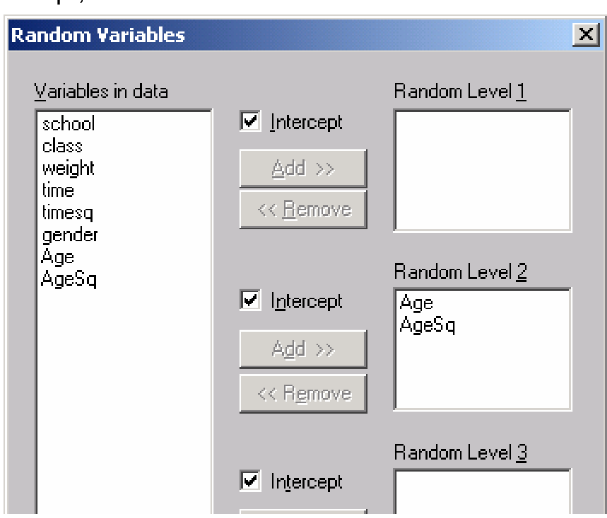
Syntax

```
RANDOMn = <list of variables names to be included as random effects on level n> ;
```

Example

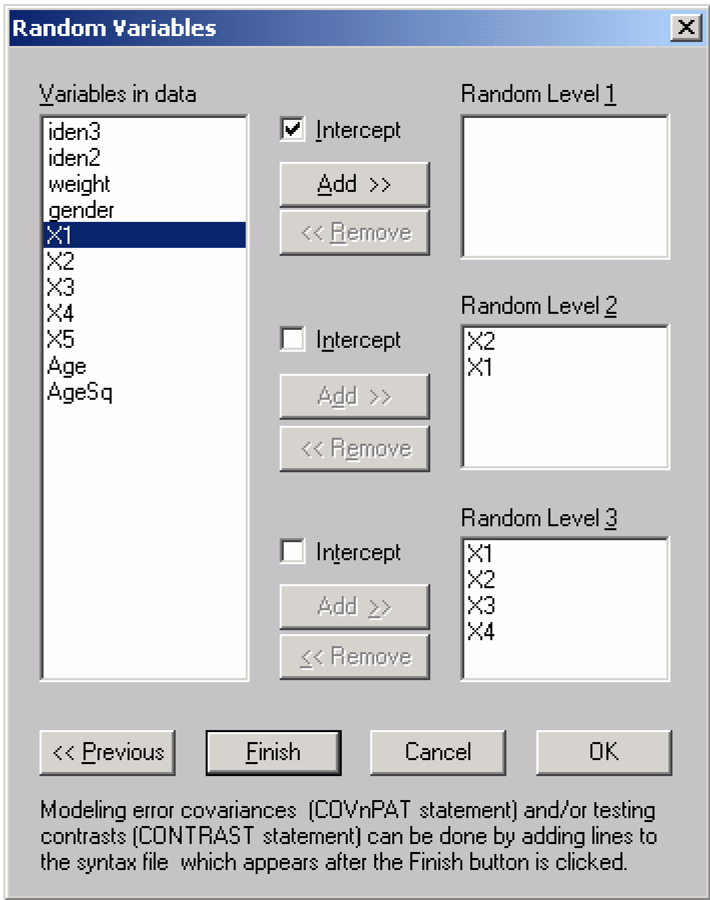
1. The **Random Variables** dialog box shown below corresponding to the commands

```
RANDOM1 = intcept;
RANDOM2 = intcept Age Agesq;
RANDOM3 = intcept;
```



2. The settings corresponding to the following commands are shown on the **Random Variables** dialog box below:

```
RANDOM3 = X1 X2 X3 X4 ;
RANDOM2 = X2 X1;
RANDOM1 = intcept;
```



From this hypothetical example the following can be seen:

- The random variables may be listed in any order.
- Any or all of the possible predictors may be included in a RANDOM command at any level of the model.

The RANDOM1 command may be omitted in the case of a multivariate model or if a model with no random component on level-1 of the hierarchy is to be fitted. Thus the following set of commands may be used:

```
ID3 = iden3;
```

```
ID2 = iden2;
RANDOM3 = X1:X4 ;
RANDOM2= X3:X4 ;
```

It is possible to place constraints on elements of the random coefficient covariance matrices. Information on the constraints permitted and on the provision of initial values for elements of these matrices are discussed elsewhere (see Sections 4.3.3 and 4.3.4 for the COVnPAT and COVnVAL commands respectively).

4.3.14 RESPONSE command

The RESPONSE command contains information on the response variable(s) to be used in the analysis. When the syntax file is created using the interface dialogs, the RESPONSE command is automatically generated. This command is entered in the **Select Response and Fixed Variables** dialog box, which follows the **Title and Options** dialog box. Since variable names are case sensitive, spelling, etc. of the names of the response variables must be the same as those used in the data spreadsheet (*.**psf** file). The RESPONSE command is a **required** command.

Syntax

```
RESPONSE = <response variable(s)>;
```

In the case of a multivariate model, more than one response variable may be listed in the RESPONSE command. Response variables may be entered in any order.

Example

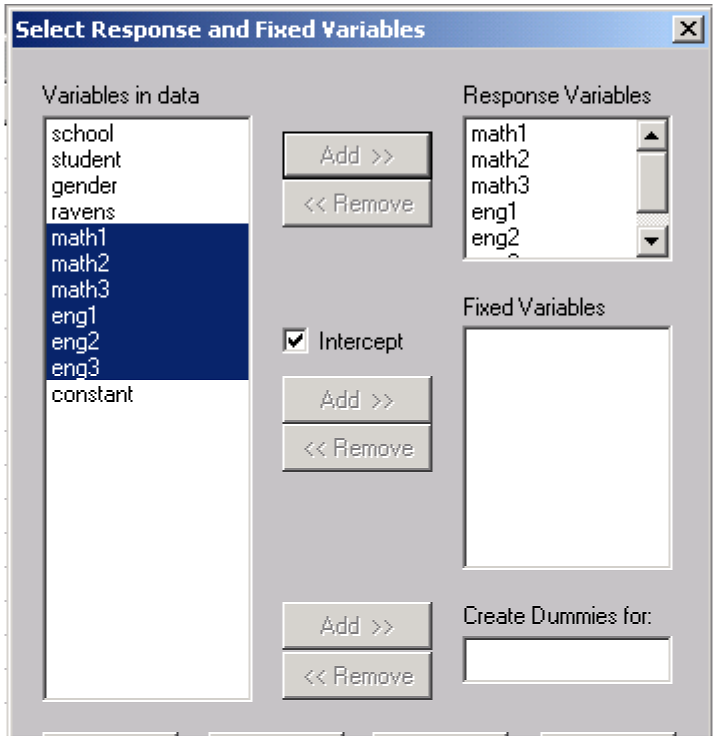
1. In the RESPONSE command below, the response variable is indicated as the variable Y1:

   ```
   RESPONSE = Y1;
   ```

2. The RESPONSE command for a multivariate model, in which 6 response variables are listed,

   ```
   RESPONSE = Math1 Math2 Math3 Eng1 Eng2 Eng3;
   ```

corresponds to the selection shown on the **Select Response and Fixed Variables** dialog box shown below.



4.3.15 SY command

The SY command is used to specify the PRELIS System File (PSF) to be analyzed, and is automatically generated if the multilevel model specifications are built via the dialog boxes. The SY command is a **required** command.

Syntax

```
SY = <filename>;
```

where <filename> denotes the complete name (including folder name) of the PSF. The folder name may be omitted if the PSF and multilevel syntax file are in the same folder.

Example

The command shown below is used to open the PSF file **kanfer.psf**, located in the **MLEVELEX** folder.

```
SY = 'C:\MLEVELEX\kanfer.psf';
```

4.3.16 TITLE command

The TITLE command allows you to provide a description of the analysis to be performed. This command, like all commands excluding the OPTIONS command, can be placed anywhere in the syntax file. When generating syntax via the interface, the TITLE command corresponds to the first entry on the **Title and Options** dialog box. The maximum permissible length of this **optional** command is 70 characters.

Syntax

TITLE = <title as provided by the user>;

Default

No title.

Example

The TITLE command shown below corresponds to the example discussed in Section 4.4.1.

TITLE = Level-3 model with design weights;

4.3.17 WEIGHTn command

The WEIGHT command is used to specify design weights for each level of the multilevel model. One WEIGHT command for each level of the hierarchy may be included in the syntax file. For a 2-level model, either or both level-1 and level-2 weights, if available, can be used. Likewise, any combination of weights can be selected for a 3-level model. Use of the command is **optional**.

Syntax

WEIGHTn = <name>;

where n denotes a positive integer, (1, 2, 3), for the weight level and <name> denotes the case sensitive name of the weight variable.

Default

No weights.

Example

The WEIGHT command shown below indicates the use of the level-1 weighting variable SPWT.

```
WEIGHT1 = SPWT;
```

4.4 Examples

The analysis of data with a hierarchical structure is known in the literature as, amongst others, hierarchical modeling, random coefficient modeling, latent curve modeling, growth curve modeling, or multilevel modeling. Here we opt to use "multilevel modeling" to describe models exhibiting nested hierarchical structures.

The basic idea is that units, be it patients or measurements, are nested within units at a higher level of the hierarchy. For example, multiple blood pressure measurements may be "nested" within patients, where patients form the next, higher, level of the hierarchy. Alternatively, duration of stay within a hospital for each individual may form measurements nested within a hospital. Here the individuals are the lower-level units, nested within the hospitals that serve as the higher-level units. No matter which of these structures applies, the outcome measured at the lowest level may be described using regression coefficients at some or all the levels of the hierarchy. Variance components at different levels of the hierarchy can be included for study. This allows the researcher to evaluate the variation in outcome at various levels of the hierarchy, while inclusion of any moderating effects is optional. In addition, the dependence of repeated measurements belonging to one experimental unit in a typical growth curve analysis, for example, is taken into account with this approach. Multilevel models are also suited to the analysis of unbalanced data, and thus estimates can be obtained for units for which a very limited amount of information is available.

Multilevel models are particularly useful in the modeling of data from complex surveys. Cluster or multi-stage samples designs are frequently used for populations with an inherent hierarchical structure. Ignoring the hierarchical structure of data has serious implications. The use of alternatives such as aggregation and disaggregation of information to another level can induce an increase in collinearity among predictors and large or biased standard errors for the estimates.

Multilevel models may be fitted to complex survey data or to data from a simple random sample by using the options on the **Multilevel Models** menu. This feature is illustrated by fitting models to both real and simulated data in the sections to follow.

4.4.1 Three-level analysis of health expenditure data

The data

The data set used here is the same as that used in Section 3.4.2, and forms part of the data library of the Medical Expenditure Panel Survey (MEPS). Collected in 1999, these data from a longitudinal national survey were used to obtain regional and national estimates of health care use and expenditure based on the health expenditures of a sample of U.S. civilian non-institutionalized participants. The survey sample design utilized stratification, clustering, multiple stages of selection, and disproportionate sampling. The sample was drawn from 143 strata, divided into 460

PSUs. Information on 23,565 participants included positive person-level weights and forms the data set used here, excluding the 1,053 participants in the original data with zero person-level weights. Data for the first 10 participants on most of the variables used in this section are shown below in the form of a LISREL spreadsheet file, named **meps2.psf**.

meps2.psf

	VARSTR99	VARPSU99	PERWT99F	TOTEXP99	RACE	SEX	INSCOV	RPOVC991	RPOVC992
1	1.000	1.000	3551.611	0.000	0.000	0.000	1.000	0.000	0.000
2	1.000	1.000	4495.334	4.205	0.000	1.000	1.000	0.000	0.000
3	1.000	1.000	7689.239	4.977	0.000	0.000	1.000	0.000	0.000
4	1.000	2.000	11069.776	5.182	0.000	1.000	0.000	1.000	0.000
5	1.000	2.000	9288.983	8.112	0.000	0.000	0.000	1.000	0.000
6	1.000	2.000	12651.611	10.090	1.000	0.000	0.000	0.000	0.000
7	1.000	2.000	6156.326	6.642	1.000	1.000	0.000	0.000	0.000
8	1.000	2.000	31359.461	8.294	0.000	0.000	0.000	1.000	0.000
9	1.000	2.000	29082.097	7.848	0.000	1.000	0.000	1.000	0.000
10	1.000	2.000	14474.962	8.471	0.000	1.000	1.000	0.000	1.000

The variables of interest are:

- VARSTR99 is the stratum identification variable (143 strata in total).
- VARPSU99 is the PSU identification variable (460 PSUs in total).
- PERWT99F represents the final sample weight, with weights ranging between 307.16 and 80061.61, correcting for both non-response and adjustments to population control totals from the Current Population Survey.
- TOTEXP99 is the natural logarithm of the total health expenditure of a respondent in 1999, ranging between 0 and 12.24 and representing actual expenditure of between $0 and $206,721.
- RACE is an ethnicity indicator, with a value of 1 indicating white respondents, and 0 denoting all other ethnic groups as well as respondents for which ethnicity is not known. This variable was recoded from the original MEPS variable RACEX.
- SEX is a gender indicator, with a value of 0 indicating a male participant and 1 a female participant; recoded from the original MEPS variable RSEX.
- INSCOV is an indicator of the level of insurance coverage, where 0 indicates private coverage any time during 1999, and 1 indicates public coverage or no insurance at all during 1999.
- RPOVC991 to RPOVC995 are five indicator variables, each associated with a category of the original MEPS variable RPOVC99 which was constructed by dividing family income by the applicable poverty line (selection of which depended on family size and composition), expressed as a percentage.

Income is a variable that is often transformed using its natural log. Doing so in effect causes the impact of each additional dollar to decrease as income increases. Logarithmic transformation is also useful in lessening the influence of outliers, as the natural logarithm of a variable is much less sensitive to extreme observations than is the variable itself.

The original MEPS variable RPOVC99 assumed a value of 1 for a family with "high" income level where family income was equal to or greater than 400% of the applicable poverty line, and a value of 2 for those with a "low income" level (associated with 125% to 200% of the poverty line). Families with "middle income", "near poor" and "negative or poor" levels of income relative to poverty line income were coded 3, 4 and 5 respectively. For the "middle income" category, the ratio (as percentage) of family income to poverty line was 200% to less than 400%. In the case of "near poor" families, the percentages ranged between 100% and 125%, and for "negative or poor", the family income was less than 100% of the relevant poverty line. Thus, a value of 1 on the indicator variable RPOVC991 indicates a family with income at the "high " level, while a value of 1 on the variable RPOVC995 indicates a family with "negative or poor" income level. The variables RPOVC992, RPOVC993, and RPOVC994 are associated with the categories "low income", "middle income" and "near poor" respectively.

Note that as each of the five indicator variables for categories of RPOVC99 is coded 1 if a participant responded in that category and 0 otherwise, only four of the five indicator variables can be used in a model where an intercept is included. Indicator variables of this type can easily be created by using the **Create Dummies for** option on the **Select Response and Fixed variables** dialog box as described in Section 4.2.4. Here, we opted to create them prior to analysis as illustration of that feature is not relevant to the example at hand.

The model

The multilevel model does not make provision for the specification of design related variables such as stratum or PSU. Instead, these design variables are used to define the hierarchical structure of the data. In this example, the stratum identification variable VARSTR99 is used as the level-3 identifier and the PSU identification variable VARPSU99 serves to identify level-2 units (*i.e.*, PSUs) nested within a given stratum. We thus use the design variables to define a three-level hierarchical structure, with participants as level-1 observations nested within PSUs, in turn nested within strata. While not explicitly acknowledging the survey design or offering a conventional design effect estimate to measure the difference in estimates obtained when implementing this design compared to estimates obtained under a simple random sample, a multilevel model offers the advantage of estimating the variation in total health care expenditure within and between PSUs.

A general three-level model for a response variable y depending on a set of r predictors $x_1, x_2, \ldots, x_r$ can be written in the form

$$y_{ijk} = \mathbf{x}'_{(f)ijk}\boldsymbol{\beta} + \mathbf{x}'_{(3)ijk}\mathbf{v}_i + \mathbf{x}'_{(2)ijk}\mathbf{u}_{ij} + \mathbf{x}'_{(1)ijk}\mathbf{e}_{ijk}$$

where $i = 1, 2, \ldots, N$ denotes the level-3 units, $j = 1, 2, \ldots, n_i$ the level-2 units, and $k = 1, 2, \ldots, n_{ij}$ the level-1 units. In this context, y_{ijk} represents the response of individual k, nested within level-2 unit j and level-3 unit i. The model shown here consists of a fixed and a random part. The fixed part of the model is represented by the vector product $\mathbf{x}'_{(f)ijk}\boldsymbol{\beta}$, where $\mathbf{x}'_{(f)ijk}$ is a typical row of the design matrix of the fixed part of the model with, as elements, a subset of the r predictors. The vector $\boldsymbol{\beta}$ contains the fixed, but unknown parameters to be estimated. The vector products $\mathbf{x}'_{(3)ijk}\mathbf{v}_i$, $\mathbf{x}'_{(2)ijk}\mathbf{u}_{ij}$, and $\mathbf{x}'_{(1)ijk}\mathbf{e}_{ijk}$ denote the random part of the model at levels 3, 2, and 1 respectively. For example, $\mathbf{x}'_{(3)ijk}$ represents a typical row of the design matrix of the random part at level 3, and $\mathbf{v}_i$ the vector of random level-3 coefficients to be estimated. The products $\mathbf{x}'_{(2)ijk}\mathbf{u}_{ij}$ and $\mathbf{x}'_{(1)ijk}\mathbf{e}_{ijk}$ serve the same purpose at levels 2 and 1 respectively. It is assumed that $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N$ are independently and identically distributed (i.i.d) with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Phi}_{(3)}$. Similarly, $\mathbf{u}_{i1}, \mathbf{u}_{i2}, \ldots, \mathbf{u}_{in_i}$ are assumed i.i.d., with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Phi}_{(2)}$, and $\mathbf{e}_{ij1}, \mathbf{e}_{ij2}, \ldots, \mathbf{e}_{ijn_{ij}}$ are assumed i.i.d., with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Phi}_{(1)}$.

Within this hierarchical framework, the model fitted to the data uses the participant's gender, ethnicity, type of health insurance cover, and measure of income relative to poverty level to predict the total expenditure on health care in 1999, the latter transformed to the natural logarithm of the actual expenses incurred.

$$\begin{aligned} \text{TOTEXP99}_{ijk} = \beta_0 + \beta_1 * \text{SEX}_{ijk} + \beta_2 * \text{RACE}_{ijk} + \beta_3 * \text{INSCOV}_{ijk} + \\ \beta_4 * \text{RPOVC991}_{ijk} + \beta_5 * \text{RPOVC992}_{ijk} + \beta_6 * \text{RPOVC993}_{ijk} + \\ \beta_7 * \text{RPOVC994}_{ijk} + v_{i0} + u_{ij0} + e_{ijk} \end{aligned}$$

where β_0 denotes the average expected total expenditure on health care in 1999, and $\beta_1, \beta_2, \ldots, \beta_7$ indicate the estimated coefficients associated with the fixed part of the model which contains the predictor variables SEX, RACE, INSCOV and the indicator variables for categories of income relative to the poverty level. The random part of the model is represented by v_{i0}, u_{ij0} and e_{ijk}, which denote the variation in average total health related expenditure over strata, between PSUs (or, in other words, over PSUs nested within strata) and between participants at the lowest level of the hierarchy.

Example: Multilevel analysis with sampling weights

Setting up the analysis

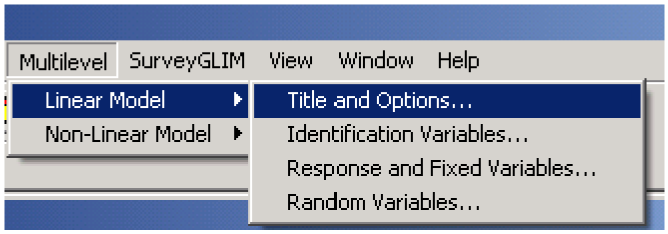
The model is fitted to the data in **meps2.psf** by using the sequence of four dialog boxes accessed via the **Multilevel**, **Linear Model** option from the main menu bar in LISREL. Note that options such as **Multilevel** and **SurveyGLIM** are only available on the main menu bar when a ***.psf** file is open.

The first step is to open the PSF shown above, which is accomplished as follows:

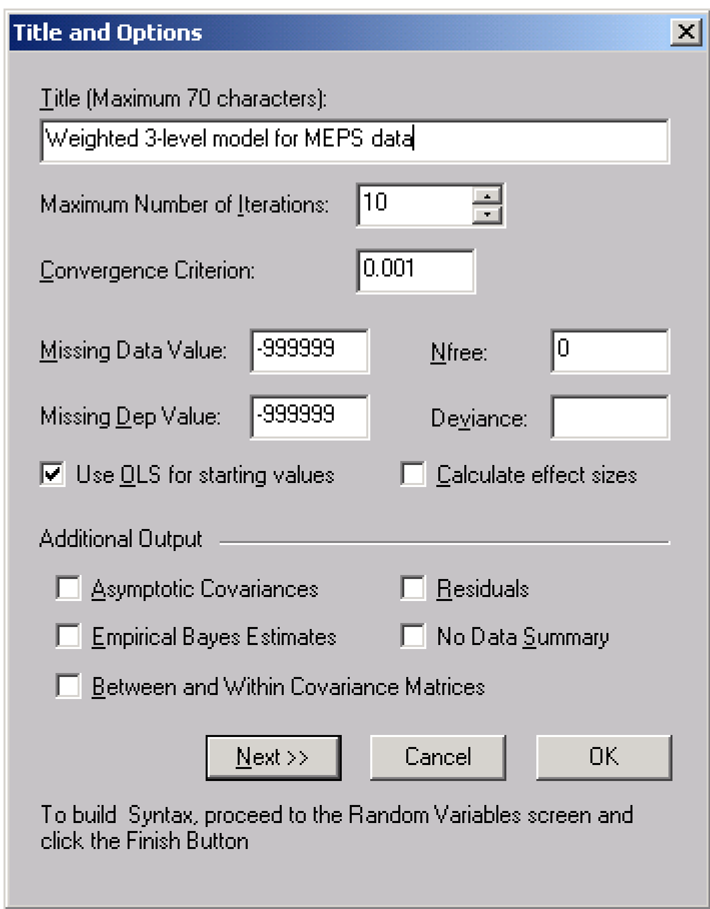
- Use the **File**, **Open** option to activate the display of an **Open** dialog box.

- Set the **Files of type** drop-down list box to **Prelis Data (*.psf)** and browse for the file **meps2.psf** in the **MLEVELEX** folder.
- Select the file and click the **Open** button to return to the main LISREL window, where the contents of the PSF are displayed.

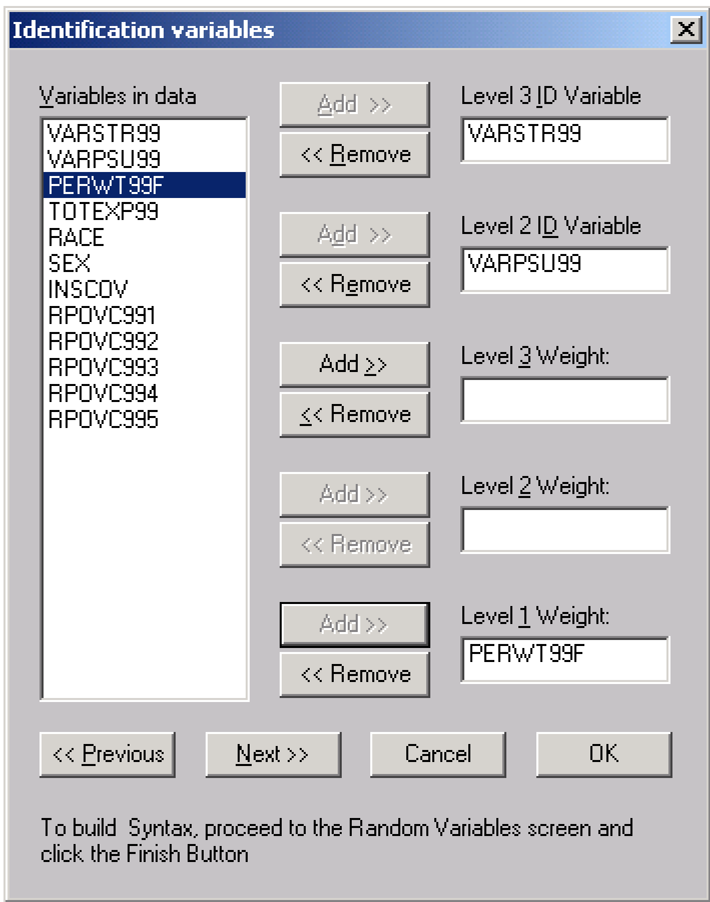
The next step is to describe the model to be fitted using the multilevel module in LISREL. From the main menu bar, select the **Multilevel** option. In this Chapter we limit our discussion to linear models, and thus the **Linear Model** option will be used throughout.



The first of the four options on the pop-up menu provide access to the **Title and Options** dialog box discussed in Section 4.2.2. Start by providing a title for the analysis in the **Title** field. In this example, default settings for all other options associated with this dialog box are used. Click the **Next** button to go to the **Identification Variables** dialog box.



On the **Identification Variables** dialog box, enter the variables defining the hierarchical structure as ID variables (see Section 4.3.9 for detailed information on the ID command). As mentioned before, the stratum identification variable is used to indicate the level-3 units in the hierarchical structure, and the PSU identification variable serves a similar purpose at level 2. Select the variables VARSTR99 and VARPSU99 as Level 3 ID variable and Level 2 ID variable respectively by clicking on the variable names in the **Variables in data** field at the left of the dialog box. Add them to the ID variable fields by clicking the appropriate **Add** button for each. This dialog box is also used to provide information on weight variables, if any. In our case, only one weight, denoted by the variable PERWT99F, is available. Select this variable from the **Variables in data** field, and add it to the **Level-1 weight** field as shown below. As all available information is now entered on this dialog box, click the **Next** button to proceed to the **Select Response and Fixed Variables** dialog box.

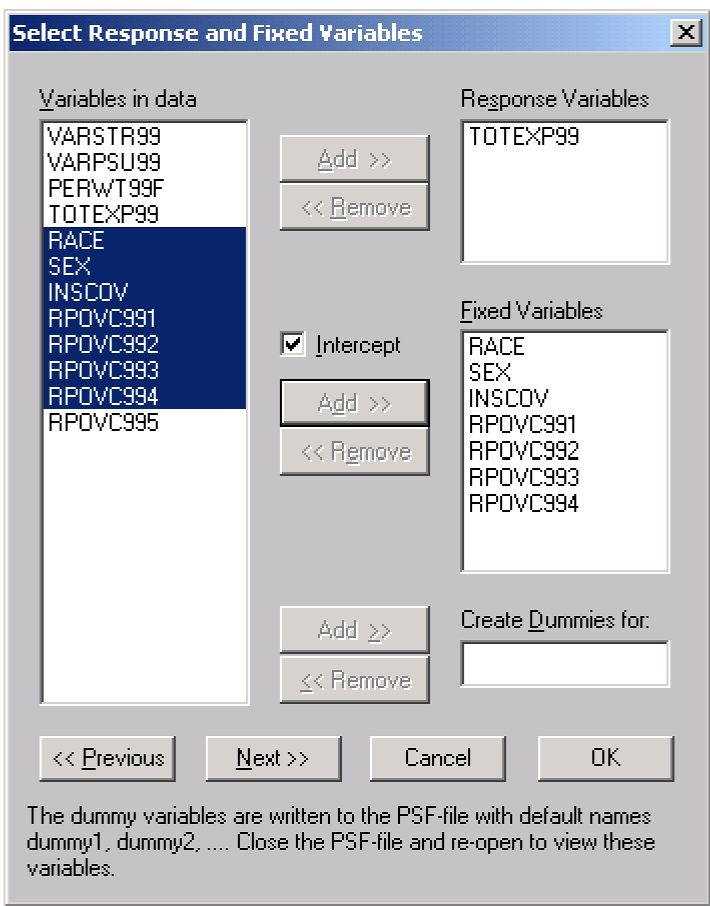


The **Select Response and Fixed Variables** dialog box, described in detail in Section 4.2.4, is used to identify the outcome variable and predictor variables, if any. Select and add the outcome variable TOTEXP99 to the **Response Variables** field in the same way as described for the previous dialog box. Next, select the variables starting from RACE to RPOVC994 by dragging the mouse over them and click the **Add** button next to the **Fixed Variables** field to include these variables as predictors in the model. This completes the specification of the response and fixed variables.

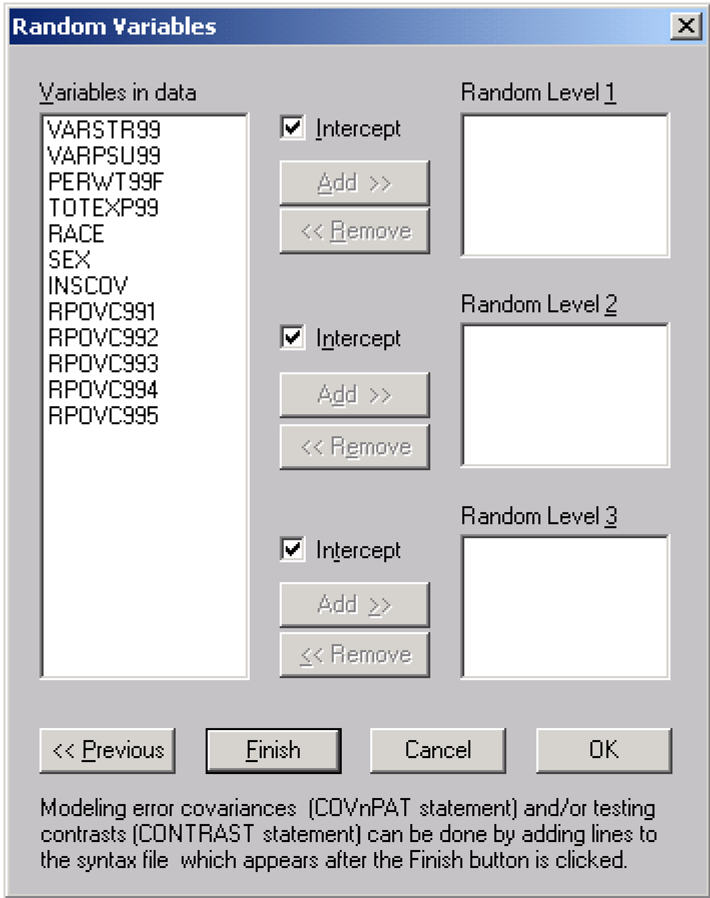
Before moving to the next dialog box, two other options available on this dialog box are worth noting.

- As previously discussed, the indicator variable associated with the highest level of income relative to the poverty line income is not selected for inclusion as the model fitted to the data has an intercept. Because of the intercept term, inclusion of all five indicator variables would lead to a design matrix of less than full rank and is bound to cause problems during the iterative procedure. An alternative approach would be to use all five indicator variables in a model without an intercept term. This can be achieved by deselecting the intercept term by unchecking the box next to Intercept.
- The **Create Dummies for** option available on the **Select Response and Fixed** dialog box can be used to create indicator variables for the categories of a categorical variable such as RPOVC99. In fact, the indicator variables RPOVC991 to RPOVC995 were created in precisely this way for inclusion in the present analysis, and simply renamed from their default names of DUMMY1 to DUMMY5 using the **Define Variable** option from the **Data** menu accessed from the main menu bar in LISREL.

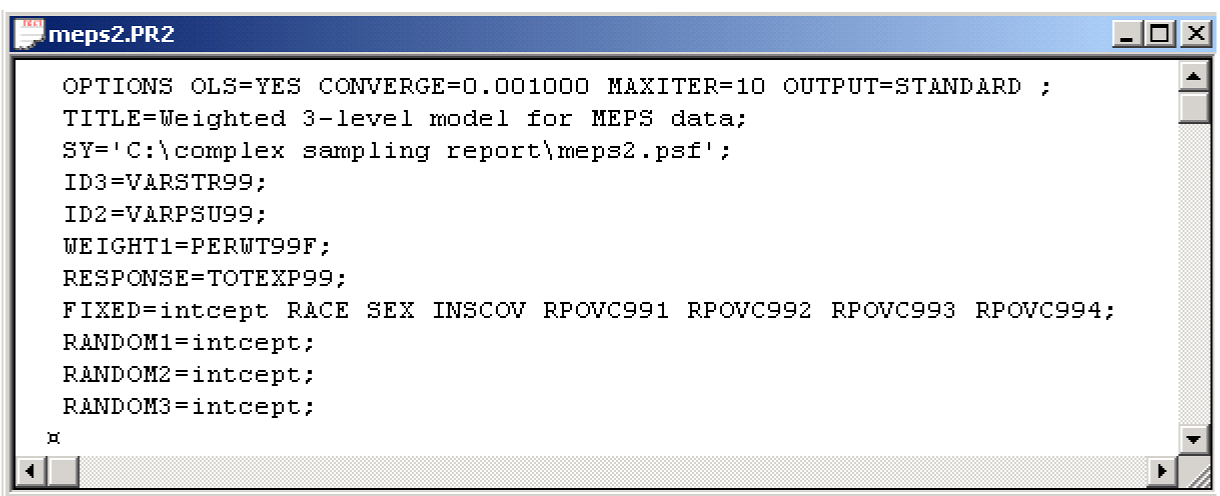
That said, proceed to the **Random Variables** dialog box by clicking the **Next** button.



The **Random Variables** dialog box shown below displays the default settings associated with this dialog box. In the current model, only the intercept coefficients are allowed to vary randomly at the various levels of the hierarchy. As this corresponds to the default settings shown on the dialog box, click the **Finish** button to generate the syntax for the model.



The syntax shown below corresponds to the information entered via the dialog boxes above. Run the model by clicking the **Run Prelis** icon on the main menu bar.




```
OPTIONS OLS=YES CONVERGE=0.001000 MAXITER=10 OUTPUT=STANDARD ;
TITLE=Weighted 3-level model for MEPS data;
SY='C:\complex sampling report\meps2.psf';
ID3=VARSTR99;
ID2=VARPSU99;
WEIGHT1=PERWT99F;
RESPONSE=TOTEXP99;
FIXED=intcept RACE SEX INSCOV RPOVC991 RPOVC992 RPOVC993 RPOVC994;
RANDOM1=intcept;
RANDOM2=intcept;
RANDOM3=intcept;
```

Discussion of results – Multilevel model with sampling weights

Portions of the output file **meps2.out** are shown below.

```
meps2.OUT

                                  +--------------+
                                  | DATA SUMMARY |
                                  +--------------+

  NUMBER OF LEVEL 3 UNITS :       143
  NUMBER OF LEVEL 2 UNITS :       460
  NUMBER OF LEVEL 1 UNITS :     23564

  ID3  :       1       2       3       4       5       6       7       8
  N2   :       2       2       2       2       2       3       2       2
  N1   :      29      85      55      86     159      48      48      78

  ID3  :       9      10      11      12      13      14      15      16
  N2   :       2       2       2      11       2       2       2       3
  N1   :     114      23      68     408      68      62     168      59

  ID3  :      17      18      19      20      21      22      23      24
  N2   :       2       5       2       2       2       2       2       2
  N1   :     364     215      69      44      40      59     171     151

  ID3  :      25      26      27      28      29      30      31      32
  N2   :       3       2       2       2       2       2      10       2
  N1   :     225      18      86      25      58      26     417      26
```

In the first section of the output file a description of the hierarchical structure is provided in the **Data Summary** section. A total of 143 strata, 460 PSUs and information from 23,564 individual participants were included at levels 3, 2 and 1 of the multilevel model. This corresponds to the survey design described earlier. In addition, a summary of the number of PSUs and participants nested within each stratum is provided. For stratum number 1 (ID3: 1), data are available from only 29 participants nested within 2 primary sampling units (N2: 2). By contrast, for stratum number 12 (ID3: 12), data are available from 408 participants (N1: 408) nested within 11 primary sampling units (N2: 11).

```
meps2.OUT

 Weighted 3-level model for MEPS data
 ITERATION NUMBER      6

                       +------------------------+
                       | FIXED PART OF MODEL    |
                       +------------------------+
------------------------------------------------------------------------------
  COEFFICIENTS              BETA-HAT      STD.ERR.      Z-VALUE       PR > |Z|
------------------------------------------------------------------------------
  intcept                    4.39123       0.11369      38.62367       0.00000
  RACE                       0.94298       0.08607      10.95549       0.00000
  SEX                        0.91057       0.03641      25.00765       0.00000
  INSCOV                    -0.65109       0.07548      -8.62642       0.00000
  RPOVC991                   0.35750       0.11408       3.13361       0.00173
  RPOVC992                  -0.13832       0.10421      -1.32736       0.18439
  RPOVC993                   0.07036       0.11668       0.60302       0.54649
  RPOVC994                  -0.32929       0.14044      -2.34470       0.01904

                       +------------------------+
                       |   -2 LOG-LIKELIHOOD    |
                       +------------------------+

  DEVIANCE= -2*LOG(LIKELIHOOD) =   118256.6158143171
  NUMBER OF FREE PARAMETERS =      11
```

The output describing the estimated **fixed effects** after convergence is shown next. The estimates are shown in the column with heading BETA-HAT, and correspond to the coefficients $\beta_0, \beta_2, \ldots, \beta_7$ in the model specification. From the z-values and associated exceedance probabilities, we see that the coefficients associated with gender, ethnicity and insurance coverage type were all highly significant. Recall that a value of 1 for the ethnicity indicator variable RACE indicated that a participant was white, with a value of 0 assigned to participants from all other ethnic groups. The positive estimated coefficient for this variable indicates an increase of 0.94298 units in the logarithm of total health expenditure, holding all other predictors constant. Similarly, female participants (coded "1" on the gender indicator SEX), are expected to have a total health expenditure 0.91057 higher than male participants if all other variables are held constant. In contrast, participants with public coverage or no coverage have a lower expected total expenditure, as indicated by the negative estimated coefficient -0.65109.

Turning to the indicator variables associated with income relative to the poverty line income, it can be seen that only two of the indicator variables, RPOVC991 and RPOVC994, have estimated coefficients that are significantly different from zero at a 5% level of significance. In the case of families with a "high" income, the estimate of 0.35750 for RPOVC991 indicates an expected increase in expenditure, while for "near poor" families, the estimate of -0.32939 indicates an expected decrease in expenditure, holding all other variables constant.

Estimated outcomes for different groups

To evaluate the expected effect of the measure of a family's income to the corresponding poverty line income, suppose that the variables RACE, SEX, and INSCOV are held at zero, as would be the case for a nonwhite male participant with private insurance coverage. If such a participant originates from a family with "high" income, the logarithm of total health expenditure is expected to be

$$\begin{aligned}
&\beta_0 + \beta_4(\text{RPOVC991}) + \beta_5(\text{RPOVC992}) + \beta_6(\text{RPOVC993}) + \beta_7(\text{RPOVC994}) \\
&= \beta_0 + \beta_4 \\
&= 4.39123 + 0.35750 \\
&= 4.74873
\end{aligned}$$

which translates to a projected total expenditure of $e^{4.74873} = \$115.437$. In contrast, for a participant with similar demographic background and coverage from a "near poor" family, we obtain a projected total expenditure of

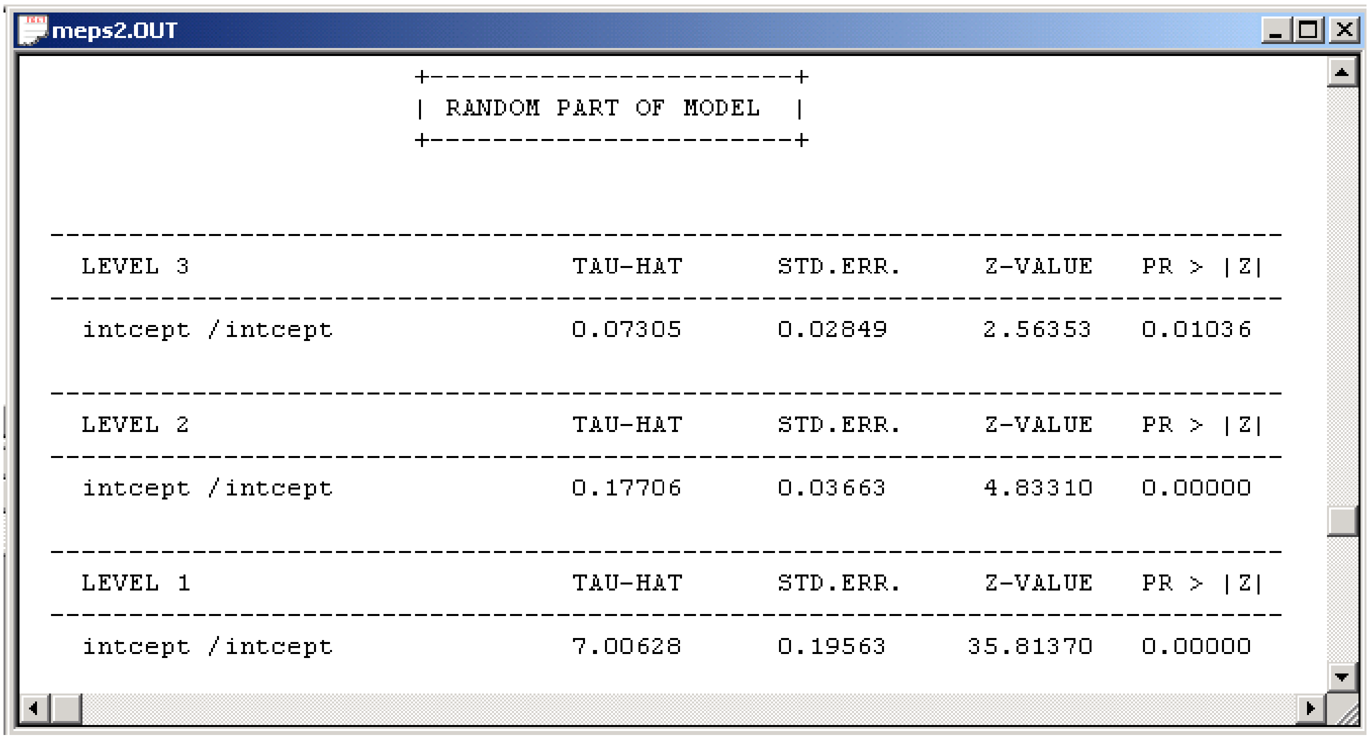
$$\begin{aligned}
&e^{\beta_0+\beta_7} \\
&= e^{4.39123-0.32929} \\
&= \$58.086
\end{aligned}$$

The predicted total expenditure (as natural logarithm) for similar participants from "low", "middle" or "negative or poor" families are similarly obtained by calculating $e^{\beta_0+\beta_5}$, $e^{\beta_0+\beta_6}$ and e^{β_0} respectively.

Table 4.1: Predicted total health expenditure for various subgroups

Respondents with high family income (RPOVC991 = 1)	**Male** (SEX = 0)		**Female** (SEX = 1)	
	Insurance coverage:		**Insurance coverage:**	
	Private (INSCOV=0)	**Public/none** (INSCOV = 1)	**Private** (INSCOV=0)	**Public/none** (INSCOV = 1)
Nonwhite (RACE = 0)	\$115	\$60	\$287	\$150
White (RACE = 1)	\$296	\$155	\$737	\$384
Respondents with near poor income (RPOVC994 = 1)				
Nonwhite (RACE = 0)	\$58	\$30	\$145	\$75
White (RACE = 1)	\$149	\$78	\$370	\$193

In Table 4.1, the predicted total health expenditure is given for respondents with high or near poor family income, for each of the subpopulations formed by gender, ethnicity and insurance coverage. For purposes of the comparison, results are expressed in U.S. dollars, rather than in the natural logarithmic units of the outcome variable TOTEXP99. Respondents from families with high income consistently outspend their near poor counterparts by approximately 100%, regardless of gender, ethnicity or level of insurance coverage. In families with high income, female respondents spent more in 1999 than their male counterparts, regardless of ethnicity. This is generally also true for near poor respondents. It is also apparent that the total health expenditure in 1999 was higher for respondents with private insurance than for respondents with public or no coverage, and that white respondents spent more than respondents from other ethnic groups, regardless of gender or the level of family income. From exploratory analyses, we know that the outcome variable TOTEXP99 is highly skewed, with median 1999 expenditure of $ 377.41. When this is taken in account, we can conclude that, generally speaking, white females spent more on health in 1999 than 50% of all respondents in the sample.


meps2.OUT

RANDOM PART OF MODEL

LEVEL 3	TAU-HAT	STD.ERR.	Z-VALUE	PR > |Z|
intcept /intcept	0.07305	0.02849	2.56353	0.01036

LEVEL 2	TAU-HAT	STD.ERR.	Z-VALUE	PR > |Z|
intcept /intcept	0.17706	0.03663	4.83310	0.00000

LEVEL 1	TAU-HAT	STD.ERR.	Z-VALUE	PR > |Z|
intcept /intcept	7.00628	0.19563	35.81370	0.00000

The output for the **random part** of the model follows, and is shown in the image above. There is significant variation in the average estimated total health expenditure at all levels, with the most variation over the participants (level-1), and the least variation over strata (level-3).

An estimate of the level-2 cluster effect, for example, is obtained as

$$\frac{0.17706}{0.07305+0.17706+7.00628}\times 100\% = 2.41\%$$

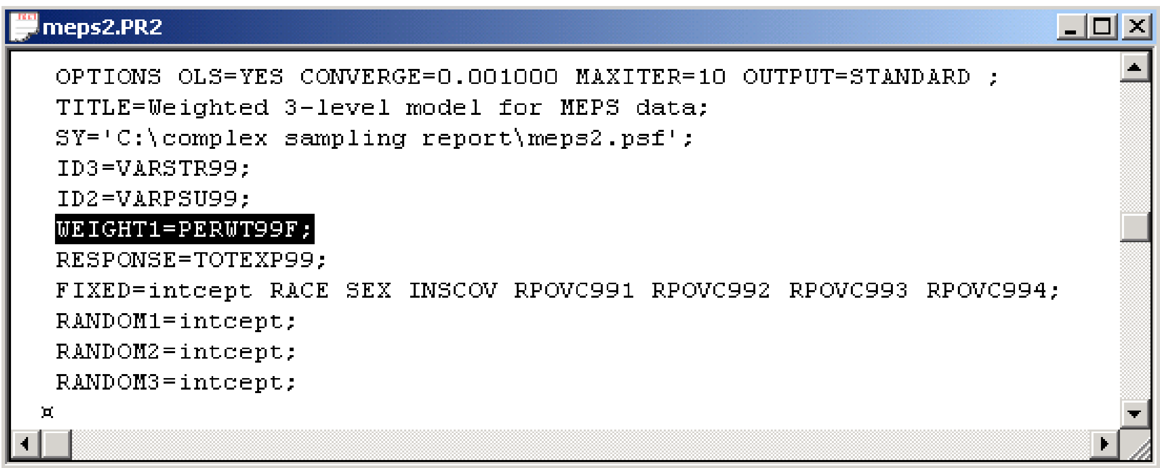
indicating that only 2.41% of the total variance explained is at level-2 of the model.

Example: Multilevel analysis without sampling weights

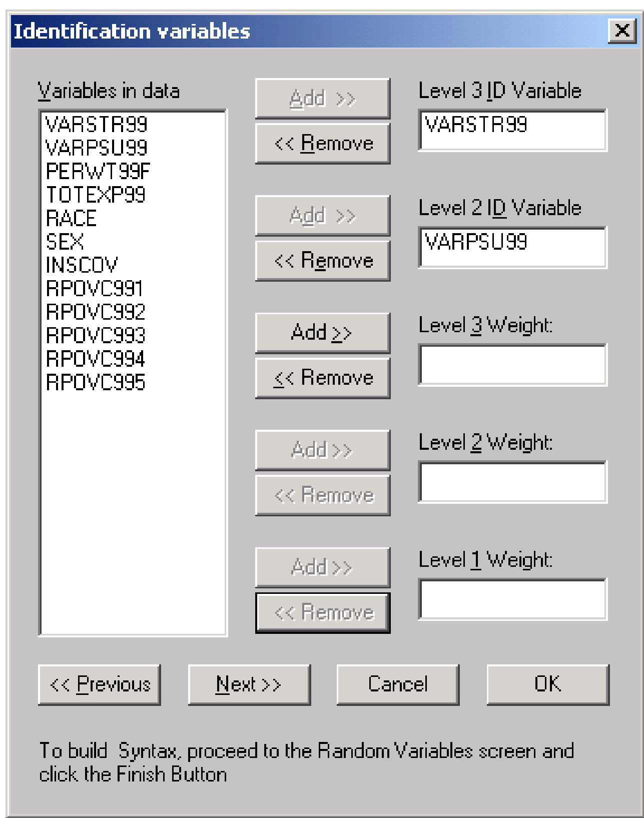
To evaluate the effect on the estimated coefficients if the sampling weights are ignored for data known to come from a disproportionally sampled survey, we fit the same model without a WEIGHT command.

Setting up the analysis

To fit the unweighted model, the syntax file from the previous analysis can be edited by simply deleting the WEIGHT1 command from the syntax file as shown below.




```
OPTIONS OLS=YES CONVERGE=0.001000 MAXITER=10 OUTPUT=STANDARD ;
TITLE=Weighted 3-level model for MEPS data;
SY='C:\complex sampling report\meps2.psf';
ID3=VARSTR99;
ID2=VARPSU99;
WEIGHT1=PERWT99F;
RESPONSE=TOTEXP99;
FIXED=intcept RACE SEX INSCOV RPOVC991 RPOVC992 RPOVC993 RPOVC994;
RANDOM1=intcept;
RANDOM2=intcept;
RANDOM3=intcept;
¤
```



Alternatively, the **Level-1 Weight** field on the **Identification Variables** dialog box can be cleared by clicking on this field and then clicking the **Remove** button next to this field.

Clicking **Next** on this and the next two dialog boxes, followed by clicking the **Finish** button on the **Random Variables** dialog box will generate a revised syntax file.

Discussion of results – Multilevel model without sampling weights

After running the analysis by clicking the **Run Prelis** icon on the main menu bar, the following output is obtained for the fixed and random parts of the unweighted model.

```
 ITERATION NUMBER     4
                      +-----------------------+
                      | FIXED PART OF MODEL   |
                      +-----------------------+

-----------------------------------------------------------------------------
  COEFFICIENTS              BETA-HAT      STD.ERR.      Z-VALUE       PR > |Z|
-----------------------------------------------------------------------------
  intcept                    4.45841       0.08350      53.39310       0.00000
  RACE                       0.68364       0.04971      13.75292       0.00000
  SEX                        0.93063       0.03581      25.98628       0.00000
  INSCOV                    -0.61785       0.04571     -13.51696       0.00000
  RPOVC991                   0.49302       0.06489       7.59791       0.00000
  RPOVC992                  -0.15390       0.06677      -2.30502       0.02117
  RPOVC993                   0.10053       0.06193       1.62342       0.10450
  RPOVC994                  -0.34592       0.08978      -3.85321       0.00012

                      +-----------------------+
                      |   -2 LOG-LIKELIHOOD   |
                      +-----------------------+
  DEVIANCE= -2*LOG(LIKELIHOOD) =  114663.1692676620
  NUMBER OF FREE PARAMETERS =     11
```

```
meps2.OUT

                        +----------------------+
                        | RANDOM PART OF MODEL  |
                        +----------------------+

 ------------------------------------------------------------------------------
  LEVEL 3                          TAU-HAT      STD.ERR.      Z-VALUE    PR > |Z|
 ------------------------------------------------------------------------------
  intcept /intcept                 0.14587      0.03648       3.99824    0.00006

 ------------------------------------------------------------------------------
  LEVEL 2                          TAU-HAT      STD.ERR.      Z-VALUE    PR > |Z|
 ------------------------------------------------------------------------------
  intcept /intcept                 0.17442      0.02767       6.30265    0.00000

 ------------------------------------------------------------------------------
  LEVEL 1                          TAU-HAT      STD.ERR.      Z-VALUE    PR > |Z|
 ------------------------------------------------------------------------------
  intcept /intcept                 7.46282      0.06938     107.56310    0.00000
```

In Table 4.2, the predicted total health expenditure is given for respondents with high or near poor family income, for each of the subpopulations formed by gender, ethnicity and insurance coverage. When compared to Table 4.1, where similar results were given for the weighted analysis, no difference in the overall pattern of expenditure is detected. Note, however, that the predicted expenditure for Nonwhite respondents (RACE = 0) are consistently higher in Table 4.2 than was the case in Table 4.1. For white respondents, the unweighted results shown in Table 4.2 are consistently lower than the corresponding results in Table 4.1. If sample weights are not used in the analysis, it may lead to a consistent, although small, overestimation of the health expenditure of nonwhite respondents, and to an underestimation of the health expenditures of their white counterparts.

Table 4.2: Predicted total health expenditure for various subgroups

Respondents with high family income (RPOVC991 = 1)	**Male** (SEX = 0)		**Female** (SEX = 1)	
	Insurance coverage:		**Insurance coverage:**	
	Private (INSCOV=0)	**Public/none** (INSCOV = 1)	**Private** (INSCOV=0)	**Public/none** (INSCOV = 1)
Nonwhite (RACE = 0)	$141	$76	$359	$193
White (RACE = 1)	$280	$151	$710	$383
Respondents with near poor income (RPOVC994 = 1)				
Nonwhite (RACE = 0)	$61	$33	$154	$83
White (RACE = 1)	$120	$65	$304	$164

Results for the two models (weighted and unweighted) are summarized in Table 4.3. While results for the models fitted in this case are not dramatically different, we observe that while some coefficients are larger for the unweighted model (for example, the estimates for intcept, SEX, INSCOV, and most markedly for RPOVC991), coefficients for RPOVC992 and RACE are larger for the weighted model. The largest difference observed is in the case of ethnicity, where an estimated increase of 0.94 in expenditure is associated with a white respondent under the weighted model, compared to only 0.68 for a white respondent in the unweighted model (holding all other variables constant). As this translates to a difference of $e^{0.26} = 1,296$ in total health expenditure for 1999, this difference is more important than it seems at first glance. In addition, the models are sufficiently different in that coefficients statistically significant in one model are no longer significant in the other, as illustrated by the estimated coefficients for the indicator variable RPOVC992. In the weighted model, low income respondents are not expected to have a significantly different expected total expenditure, while the estimated coefficient under the unweighted model indicates a statistically significant decrease of -0.15 units in the total expected expenditure.

Table 4.3: Results of weighted and unweighted level-3 models for the MEPS data

Coefficient	**Estimate (weighted)**	**Estimate (unweighted)**
intcept	4.39123	4.45841
RACE	0.94298	0.68364
SEX	0.91057	0.93063
INSCOV	-0.65109	-0.61785
RPOVC991	0.35750	0.49302
RPOVC992	-0.13832*	-0.15390
RPOVC993	0.07036*	0.10053*
RPOVC994	-0.32929	-0.34592
Level-1 variance	7.00628	7.46282
Level-2 variance	0.17706	0.17442
Level-3 variance	0.07305	0.14587

* Not significant at a 5% level of significance.

Comparison with SurveyGLIM model

A similar model was fitted to the data using the SurveyGLIM module (see Section 3.2.1) and a Normal-Identity model. Results are summarized in Table 4.4. In general, results obtained for the two models are similar.

Table 4.4: Results of weighted multilevel and SurveyGLIM models for the MEPS data

Coefficient	Multilevel model	SurveyGLIM model
intcept	4.39123	4.2771
RACE	0.94298	0.9393
SEX	0.91057	0.9204
INSCOV	-0.65109	-0.6952
RPOVC991	0.35750	0.4319
RPOVC992	-0.13832*	-0.1415*
RPOVC993	0.07036*	0.1186*
RPOVC994	-0.32929	-0.3433

* Not significant at a 5% level of significance.

We conclude that, where weight variables are available for survey data, these should be included in the model as neglecting to do so can have a definite impact on the estimated coefficients. In the current example, results for the two models were not dramatically different, but comparison of predicted expenditure indicated the risk of consistently over- or underestimating the total health expenditure for groups with different levels of family poverty. From the results it seems reasonable to assume that it included a component to adjust for the over/undersampling of ethnic and gender groups, a procedure commonly used in survey design to ensure representativeness. This is in agreement with the fact that, according to the MEPS HC-054: 1999 report, Hispanic and black households were oversampled at rates of approximately 2 and 1.5 times the rate of remaining households.

4.4.2 Three-level analysis of simulated data

Unlike real data sets, simulated data sets have the advantage that the true population parameters are known. Consequently, it is possible to evaluate how closely a particular model approaches these values.

The data

A linear growth curve model with two dummy-coded covariates (Lang1 and Lang2) is fitted to a simulated dataset **surveyhlm.psf** in the **MLEVELEX** folder. It is assumed that the level-3 units are institutions. Within each of 100 institutions, 10 patients are selected on the basis of their initial achievement in a test of short term memory (Score1) and measurements were repeated over six time intervals for five patients from each institution and over 4 time intervals for the remaining 5. In the table below, (Weight3) shows the level-3 weight calculations based on standardized initial scores. See Section 4.5.3 for additional information on the weight calculations.

```
Interval  Lower      Upper    % Expected  % Selected   Weight3
--------------------------------------------------------------
 1        -Inf       -1.00     15.87       10.00        1.587
 2        -1.00      -0.70      8.33       10.00        0.833
 3        -0.70      -0.20     17.88       10.00        1.788
 4        -0.20       0.00      7.93       10.00        0.793
 5         0.00       0.30     11.79       10.00        1.179
 6         0.30       1.00     22.34       10.00        2.234
 7         1.00       1.30      6.19       10.00        0.619
 8         1.30       1.80      6.09       10.00        0.609
 9         1.80       2.30      2.52       10.00        0.252
10         2.30       Inf       1.07       10.00        0.107
```

Ten patients were selected from each institution as follows:

- Four from ethnic group 1 with Weight2 = 7.0/4.0
- Three from ethnic group 2 with Weight2 = 2.0/3.0
- Three from ethnic group 3 with Weight2 = 1.0/3.0

The first 10 records of the dataset in **surveyhlm.psf** are shown below.

surveyhlm.psf

	Institut	Patient	Score	Time	Lang1	Lang2	WT3	WT2
1	1.00	1.00	-1.84	0.00	0.00	0.00	1.59	1.75
2	1.00	1.00	-0.89	1.00	0.00	0.00	1.59	1.75
3	1.00	1.00	-1.21	2.00	0.00	0.00	1.59	1.75
4	1.00	1.00	-3.24	3.00	0.00	0.00	1.59	1.75
5	1.00	1.00	-1.16	4.00	0.00	0.00	1.59	1.75
6	1.00	1.00	-1.30	5.00	0.00	0.00	1.59	1.75
7	1.00	2.00	-0.07	0.00	0.00	0.00	1.59	1.75
8	1.00	2.00	2.99	1.00	0.00	0.00	1.59	1.75
9	1.00	2.00	0.92	2.00	0.00	0.00	1.59	1.75
	1.00	2.00	3.63	3.00	0.00	0.00	1.59	1.75

Note that the data were simulated in such a way that odd-numbered patients have six score measurements at time points 0, 1, 2, 3, 4, 5. The even-numbered patients have only four score measurements.

The model

The three-level model used here is similar to that described in the previous section. Within that hierarchical framework, 500 data sets, **surveyhlm.psf** being the first, were simulated (see Section 4.5.3) according to the following hypothetical model

$$Score_{ijk} = \beta_0 + \beta_1 * Time + \gamma_1 * Lang1 + \gamma_2 * Lang2$$
$$+v_{i0} + Time * v_{i1} + u_{ij0} + Time * u_{ij1} + e_{ijk}$$

where i denotes institution i, (i = 1, 2, …, 100), ij patient j (j =1, 2, …, 10) in institution i and ijk the k-th measurement (k = 1, 2, …, 6) on patient j in institution i. The outcome variable Score denotes a patient's measurement on some test of interest, Time the time of measurement, and Lang1 and Lang2 are indicator variables indicating a patient's first or home language as being English or another language.

In this model, β_0 denotes the average expected score, while β_1 indicates the estimated coefficients associated with the time of measurement as represented by the fixed effect Time. The fixed part of the model also includes the predictor variables Lang1 and Lang2. The random part of the model is represented by v_{i0}, u_{ij0} and e_{ijk}, which denote the variation in score over institutions, between patients (or, in other words, over patients nested within institutions) and between measurements at the lowest level of the hierarchy.

The data were simulated under the assumption that

$$\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 1.0 \\ 0.5 \end{pmatrix}, \quad \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ -1.0 \end{pmatrix}$$

$$\phi_2 = Cov(u_{ij0}, u_{ij1}) = \begin{pmatrix} 1 & \\ 0.3 & 0.2 \end{pmatrix}$$

$$\phi_3 = Cov(v_{i0}, v_{i1}) = \begin{pmatrix} 1 & \\ 0.3 & 0.2 \end{pmatrix}$$

and

$$\sigma^2 = Var(e_{ijk}) = 1.0.$$

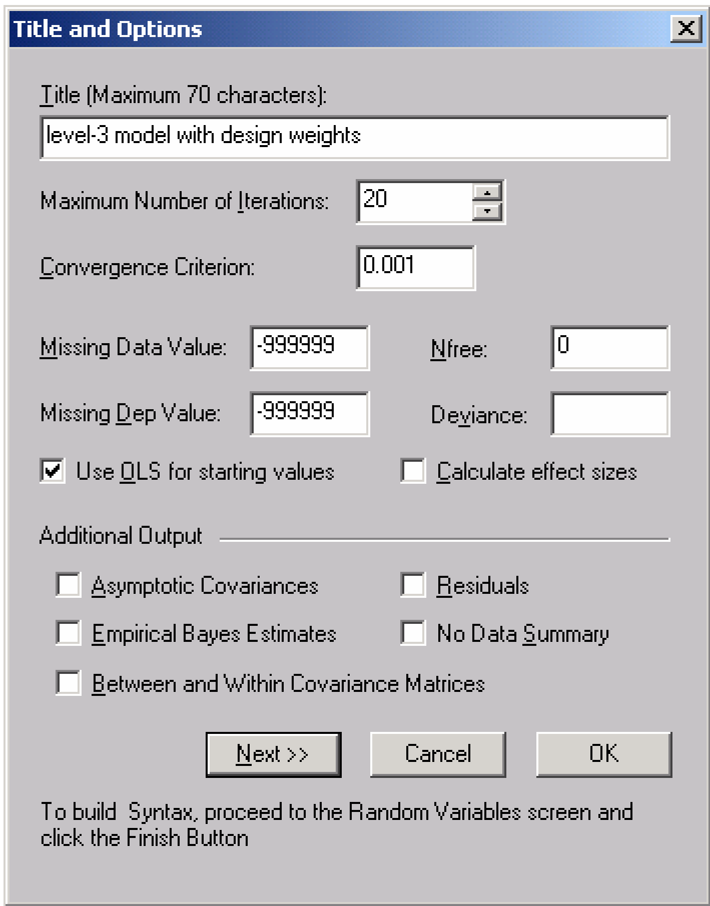
Fitting the model

The first step is to open the PSF shown above, which is accomplished as follows:

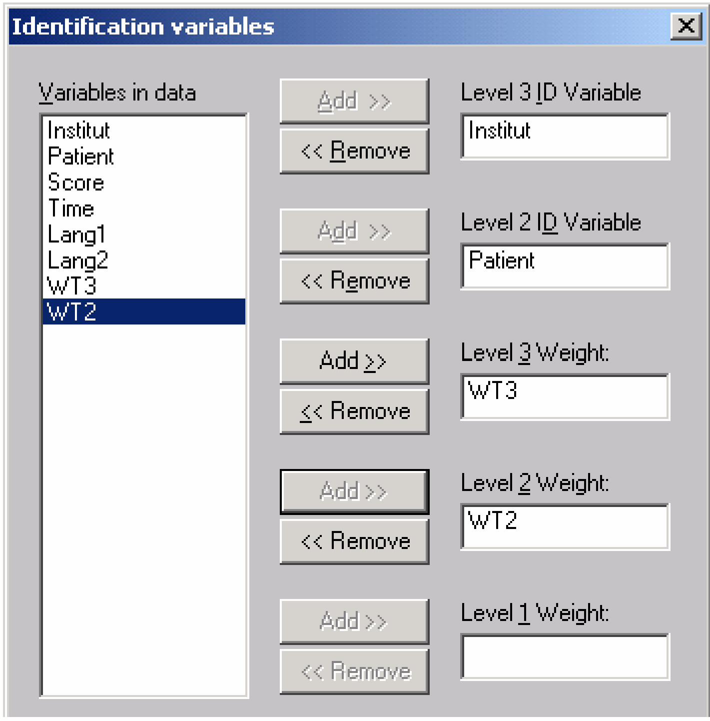
- Use the **File**, **Open** option to activate the display of an **Open** dialog box.
- Set the **Files of type** drop-down list box to **Prelis Data (*.psf)** and browse for the file **surveyhlm.psf** in the **MLEVELEX** folder.

- Select the file and click the **Open** button to return to the main LISREL window, where the contents of the PSF are displayed.

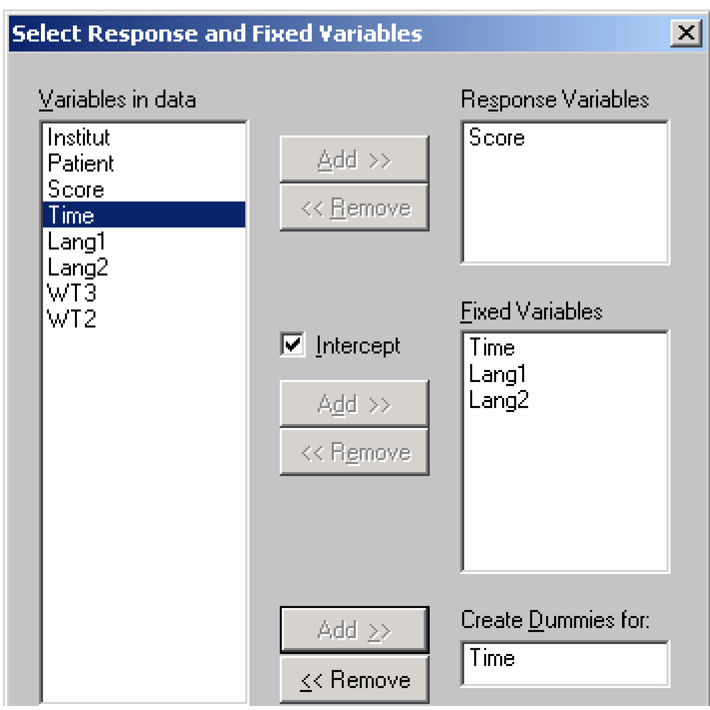
The next step is to describe the model to be fitted using the Multilevel module in LISREL. To fit a growth model to the data, we proceed as follows. From the main menu bar, select **Multilevel, Linear Model, Title and Options**. Type in a title, and change the number of iterations to 20 and the convergence criterion to 0.0001 as shown below. Click the **Next** button to activate the **Identification Variables** dialog box.



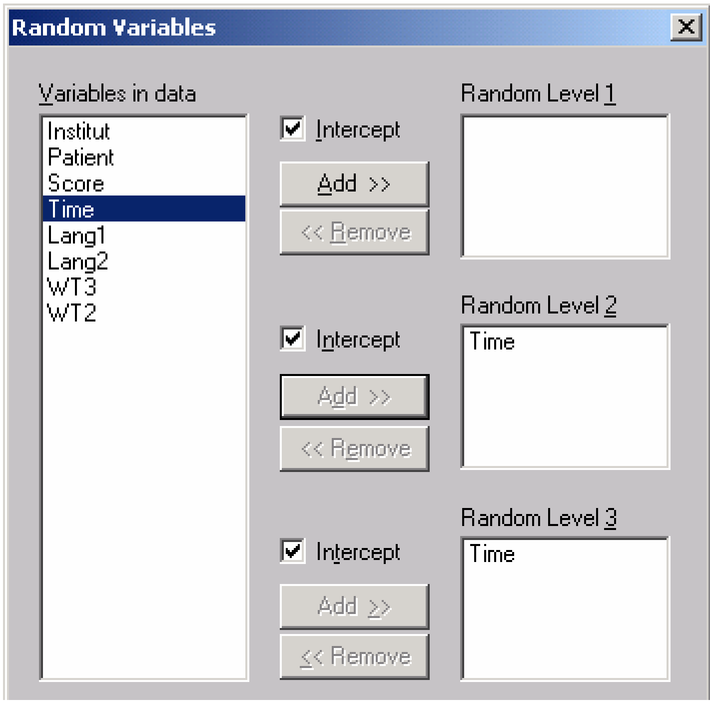
Add the level-3 and level-2 identification variables (Institut and Patient) and also the variables WT3 and WT2 as the level-3 and level-2 weights respectively. To continue to the **Response and Fixed variables** dialog box click **Next**.



Select Score as the dependent (response) variable and Time, Lang1 and Lang2 as the fixed variables (predictors). Note that an intercept term is automatically included unless the **Intercept** check box is unchecked. For illustrative purposes, Time was added to the **Create Dummies for:** text box.



Click **Next** to go to the **Random Variables** dialog box, and add Time as level-2 and level-3 random components (the variances are denoted by $Var(u_{ij1}) = \phi_{(2),22}$ and $Var(v_{i1}) = \phi_{(3),22}$ respectively. Note that by default, intercept terms are included at the different levels of the hierarchy. The level-1, level-2 and level-3 variance components for the intercept are denoted by σ_e^2, $\phi_{(2),11}$ and $\phi_{(3),11}$ respectively.



When done, click the **Finish** button to obtain the PRELIS syntax file **surveyhlm.pr2**. Click the **Run PRELIS** icon button to invoke the multilevel module.

surveyhlm.PR2

```
OPTIONS OLS=YES CONVERGE=0.001000 MAXITER=20 OUTPUT=STANDARD ;
TITLE=level-3 model with design weights;
SY='C:\Lisrel87\MLEVELEX\surveyhlm.psf';
ID3=Institut;
ID2=Patient;
WEIGHT3=WT3;
WEIGHT2=WT2;
RESPONSE=Score;
FIXED=intcept Time Lang1  Lang2;
DUMMY=Time;
RANDOM1=intcept;
RANDOM2=intcept Time;
RANDOM3=intcept Time;
```

Discussion of results

The output for the **fixed part** of the model is given first, as shown below.

```
surveyhlm.OUT

 level-3 model with design weights

 ITERATION NUMBER     4


                      +-----------------------+
                      | FIXED PART OF MODEL   |
                      +-----------------------+


------------------------------------------------------------------------------
 COEFFICIENTS              BETA-HAT      STD.ERR.      Z-VALUE       PR > |Z|
------------------------------------------------------------------------------
 intcept                    0.92865       0.11624       7.98895       0.00000
 Time                       0.51411       0.04700      10.93934       0.00000
 Lang1                      0.39340       0.10188       3.86158       0.00011
 Lang2                     -1.03430       0.12628      -8.19033       0.00000


                      +-----------------------+
                      |   -2 LOG-LIKELIHOOD   |
                      +-----------------------+

 DEVIANCE= -2*LOG(LIKELIHOOD) =  20291.45600144901
 NUMBER OF FREE PARAMETERS =     11

 CHI-SQUARE SCALE FACTOR =      0.68008
```

Recall that the "true" values of the intcept, Time, Lang1 and Lang2 parameters were 1.0, 0.5, 0.5, and -1.0 respectively. To obtain 95% confidence intervals for these estimates, we calculate

$$Estimate \pm 1.96(std.\,error)$$

and find that the confidence intervals for the estimated intcept, Time, Lang1 and Lang2 parameters are (0.7009; 1.1565), (0.4220; 0.6062), (0.1937; 0.5931) and (-1.2818; -0.7868) respectively. In all four cases, the confidence intervals include the "true" values of corresponding parameter.

Note that a χ^2 scale factor of 0.68008 is reported. This value is used to obtain a corrected χ^2-statistic for testing one model against another model, as will be shown in the next example.

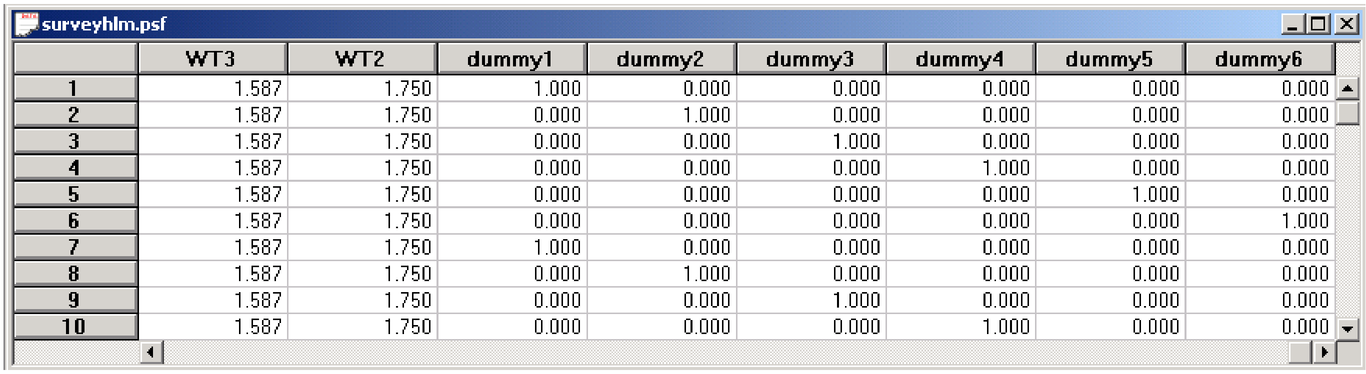
The output for the **random part** of the model is given next. Note that the parameter estimates reported in the output are generally close to the population values which were used to simulate the data. The "true" values for both the level-3 and level-2 variance-covariance components are 1.0, 0.3, and 0.2 respectively. The standard error estimates shown have been corrected as described in the theoretical section (see Section 4.6).

surveyhlm.OUT

```
                        +-----------------------+
                        | RANDOM PART OF MODEL  |
                        +-----------------------+


 --------------------------------------------------------------------------------
  LEVEL 3                         TAU-HAT      STD.ERR.      Z-VALUE    PR > |Z|
 --------------------------------------------------------------------------------
  intcept /intcept                0.93174      0.16915       5.50824    0.00000
  Time    /intcept                0.25568      0.06072       4.21048    0.00003
  Time    /Time                   0.17510      0.03408       5.13730    0.00000

 --------------------------------------------------------------------------------
  LEVEL 2                         TAU-HAT      STD.ERR.      Z-VALUE    PR > |Z|
 --------------------------------------------------------------------------------
  intcept /intcept                0.96301      0.12059       7.98585    0.00000
  Time    /intcept                0.36079      0.04117       8.76381    0.00000
  Time    /Time                   0.20039      0.02207       9.07935    0.00000

 --------------------------------------------------------------------------------
  LEVEL 1                         TAU-HAT      STD.ERR.      Z-VALUE    PR > |Z|
 --------------------------------------------------------------------------------
  intcept /intcept                1.02326      0.06542      15.64041    0.00000
```

For the level-3 variance components, 95% confidence intervals can be obtained as shown previously. The confidence intervals corresponding to intcept/intcept, Time/intcept, and Time/Time are (0.6002; 1.2633), (0.1367; 0.3747) and (0.1083; 0.2419) respectively. Again, the "true" values fall within these intervals. This conclusion also holds for confidence intervals for the level-1 and level-2 variance-covariance components, which are calculated in the same way.

Note that the spreadsheet presentation of **surveyhlm.psf** will only show the variables Institut, Patient, …, WT2, although dummy variables corresponding to the six measurement occasions were written to the actual PSF file. To see these dummy variables, close the PSF file (**without** saving it) and then use **File**, **Open** to display the modified PSF file.



surveyhlm.psf

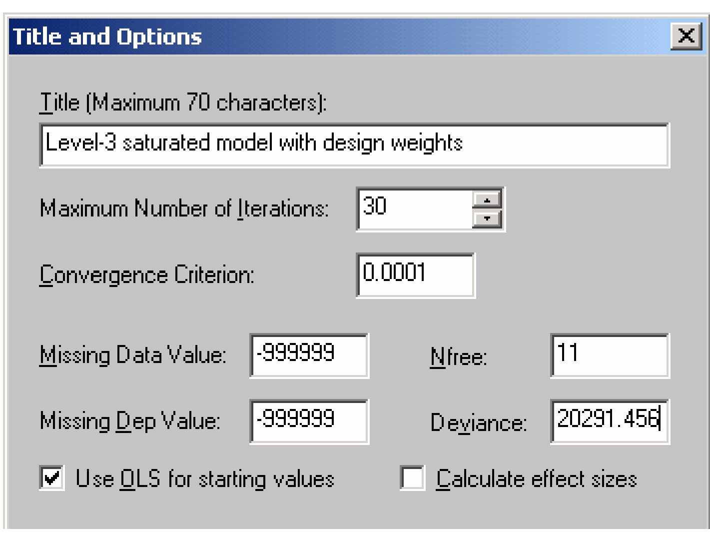
	WT3	WT2	dummy1	dummy2	dummy3	dummy4	dummy5	dummy6
1	1.587	1.750	1.000	0.000	0.000	0.000	0.000	0.000
2	1.587	1.750	0.000	1.000	0.000	0.000	0.000	0.000
3	1.587	1.750	0.000	0.000	1.000	0.000	0.000	0.000
4	1.587	1.750	0.000	0.000	0.000	1.000	0.000	0.000
5	1.587	1.750	0.000	0.000	0.000	0.000	1.000	0.000
6	1.587	1.750	0.000	0.000	0.000	0.000	0.000	1.000
7	1.587	1.750	1.000	0.000	0.000	0.000	0.000	0.000
8	1.587	1.750	0.000	1.000	0.000	0.000	0.000	0.000
9	1.587	1.750	0.000	0.000	1.000	0.000	0.000	0.000
10	1.587	1.750	0.000	0.000	0.000	1.000	0.000	0.000

4.4.3 Three-level saturated model for simulated data

Using the same simulated data described in Section 4.4.2, a "saturated model" using the dummy variables created previously is now fitted to the data. This model illustrates the use of the NFREE and DEVIANCE keywords to obtain a chi-square statistic for testing two nested models. A model of particular interest is the so-called saturated model, which is obtained by estimating the population means and the covariance matrices for both level-3 and level-2 at the six measurement occasions. The dummy variables created in the previous section, each corresponding to a specific measurement occasion, are used for this purpose.

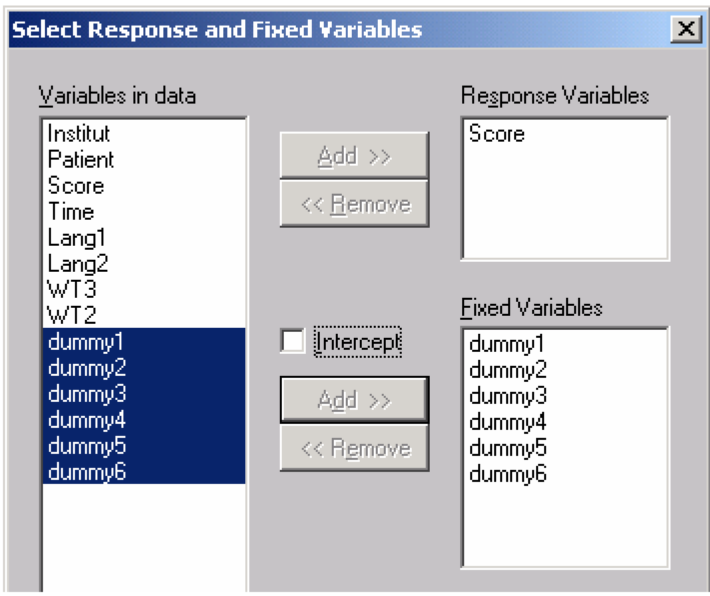
Fitting the model

The model is fitted using the same sequence of dialog boxes shown in the previous Section. In order to compare the fit of the saturated model with the fit of the model described in Section 4.4.2, the deviance statistic and number of estimated parameters from the first model are used. In the previous model, 11 parameters (4 fixed and 7 random) were estimated and a deviance statistic (-2 logL) of 20291.456 was obtained. Enter these values in the **Nfree** and **Deviance** fields of the **Title and Options** dialog box . Click **Next** to display the **Identification Variables** dialog box.

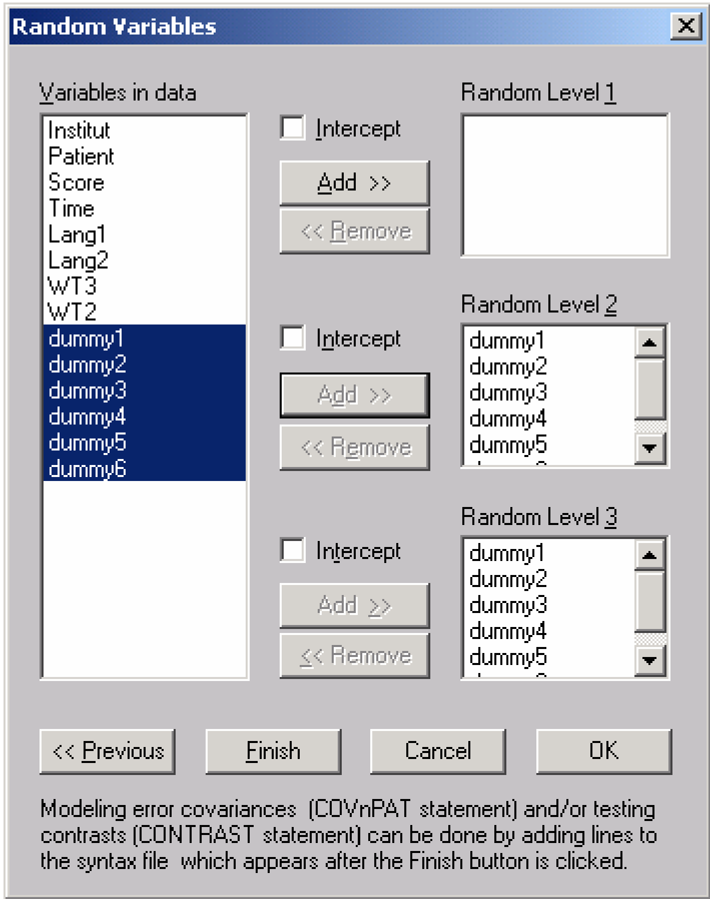


As no changes to the hierarchical structure or weight specification entered previously on the **Identification Variables** dialog box, click **Next** to load the **Response and Fixed Variables** dialog box.

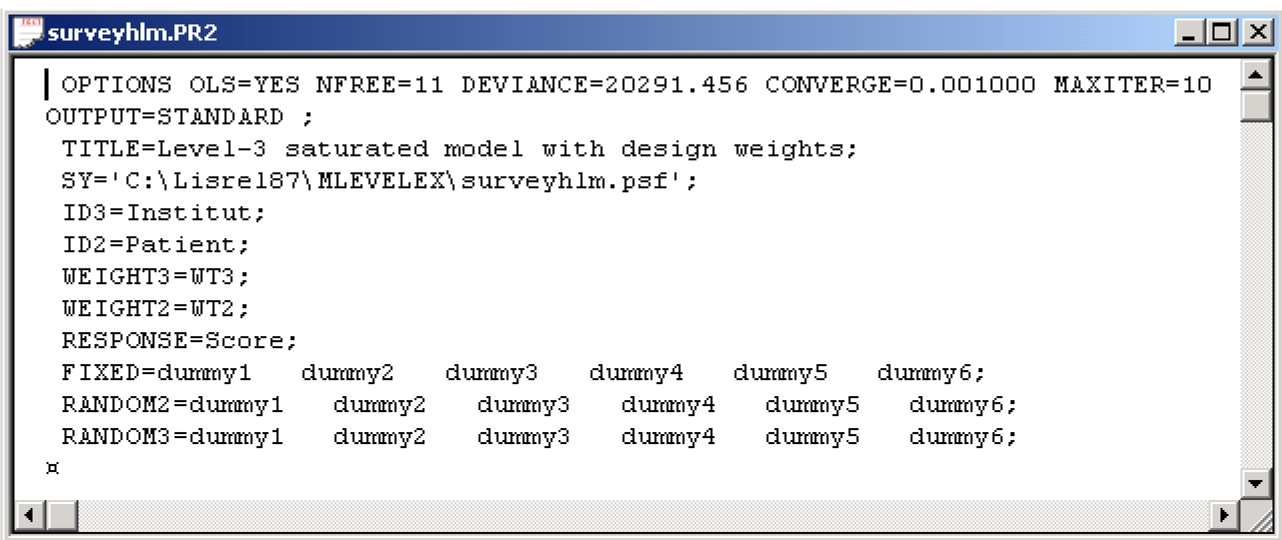
Note that one cannot add an intercept term to the fixed part of the model when dummy1 to dummy6 are selected as predictors. If the intercept term is not unchecked, then the fixed parameter coefficients can not be estimated, since the fixed-effect design matrix containing intcept, dummy1, …, dummy6 will not be of full rank. Click **Next** to proceed to the **Random Variables** dialog box.



On the **Random Variables** dialog box, the intercept terms for the random effects are unchecked and dummy variables one to six are only added at levels 2 and 3.



Click the **Finish** button to produce the PRELIS syntax file (which was subsequently saved as **surveyhlm2.pr2**).



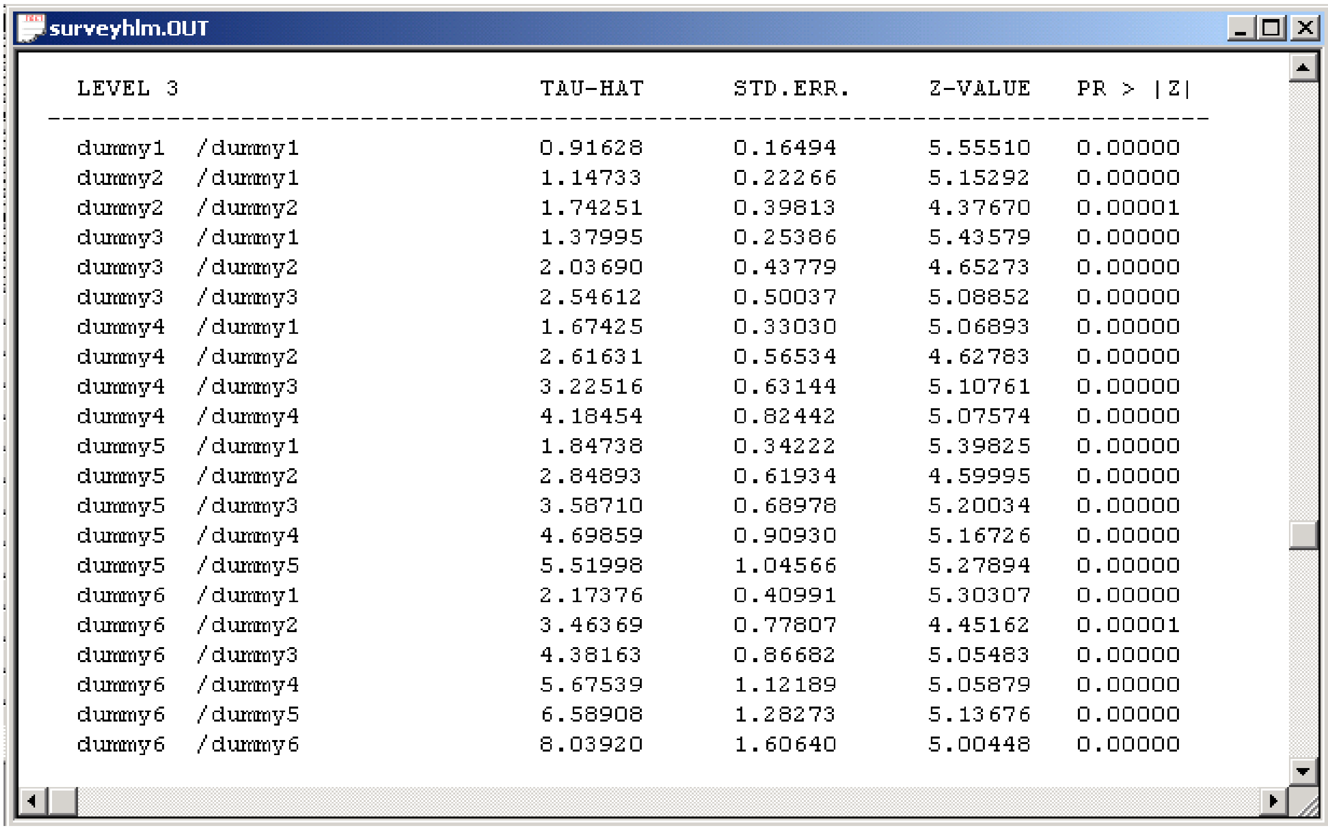

```
OPTIONS OLS=YES NFREE=11 DEVIANCE=20291.456 CONVERGE=0.001000 MAXITER=10
OUTPUT=STANDARD ;
 TITLE=Level-3 saturated model with design weights;
 SY='C:\Lisrel87\MLEVELEX\surveyhlm.psf';
 ID3=Institut;
 ID2=Patient;
 WEIGHT3=WT3;
 WEIGHT2=WT2;
 RESPONSE=Score;
 FIXED=dummy1   dummy2   dummy3   dummy4   dummy5   dummy6;
 RANDOM2=dummy1   dummy2   dummy3   dummy4   dummy5   dummy6;
 RANDOM3=dummy1   dummy2   dummy3   dummy4   dummy5   dummy6;
¤
```

Discussion of results

The portions of the output below summarize the estimated parameter values for the fixed part of the model and the goodness of fit χ^2 statistic. The $\hat{\beta}$ values are estimates of the population mean scores at each of the six measurement occasions, after controlling for the within institution and within patient variation. Note that the difference in the -2log(L) values is 2.335. The χ^2 value of 1.5565 was obtained by multiplying 2.335 with the scale factor obtained when design weights are included.

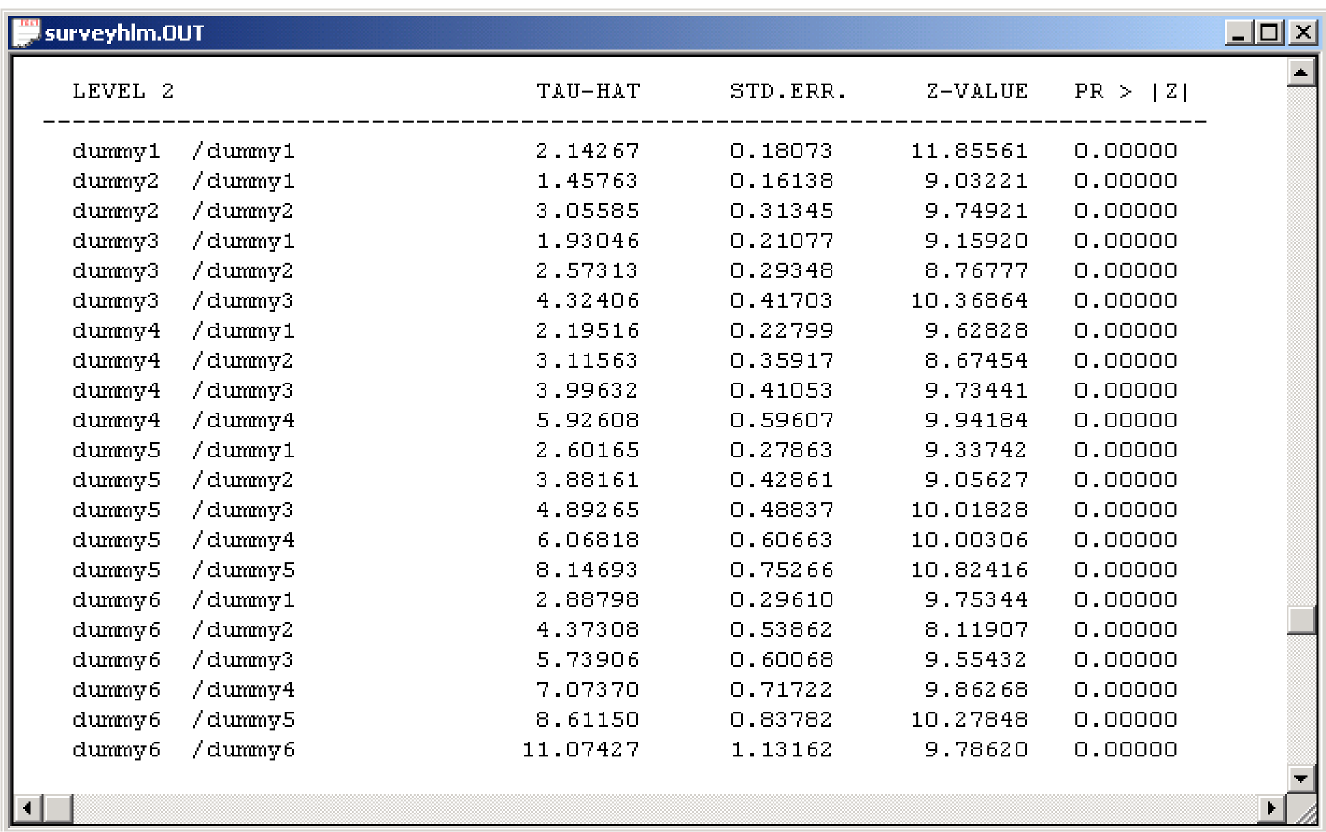
```
                        +-----------------------+
                        | FIXED PART OF MODEL   |
                        +-----------------------+
------------------------------------------------------------------------------
  COEFFICIENTS              BETA-HAT      STD.ERR.      Z-VALUE       PR > |Z|
------------------------------------------------------------------------------
  dummy1                    0.86268       0.11765       7.33286       0.00000
  dummy2                    1.47507       0.16289       9.05572       0.00000
  dummy3                    1.92424       0.19218      10.01261       0.00000
  dummy4                    2.43611       0.24712       9.85820       0.00000
  dummy5                    2.98650       0.29087      10.26745       0.00000
  dummy6                    3.40673       0.34567       9.85532       0.00000

                        +-----------------------+
                        |   -2 LOG-LIKELIHOOD   |
                        +-----------------------+

  DEVIANCE= -2*LOG(LIKELIHOOD) =  20289.12090921827
  NUMBER OF FREE PARAMETERS =     48
  CHI-SQUARE SCALE FACTOR =      0.66657

       |-----------------------------------------------|
       | Chi-square Statistic for Testing the Fit of   |
       | the Current Model versus an Alternative Model |
       |-----------------------------------------------|

      -2Log(L)=      20289.1209 with     48  Free Parameters (Current Model)
      -2Log(L)=      20291.4560 with     11  Free Parameters (Alternative Model)
       Chi-Square=       1.5565, df=     37, p-value=  1.00000
```

Results for the variance components (random part of the model) are shown below. The $\hat{\tau}$ values are estimates of the population variances/covariances at level-3 (institutions) and level-2 (patients). An inspection of the output shows that, in general, there is greater variation in scores at each time point within patients than is the case within institutions.



surveyhlm.OUT

LEVEL 3		TAU-HAT	STD.ERR.	Z-VALUE	PR > |Z|
dummy1	/dummy1	0.91628	0.16494	5.55510	0.00000
dummy2	/dummy1	1.14733	0.22266	5.15292	0.00000
dummy2	/dummy2	1.74251	0.39813	4.37670	0.00001
dummy3	/dummy1	1.37995	0.25386	5.43579	0.00000
dummy3	/dummy2	2.03690	0.43779	4.65273	0.00000
dummy3	/dummy3	2.54612	0.50037	5.08852	0.00000
dummy4	/dummy1	1.67425	0.33030	5.06893	0.00000
dummy4	/dummy2	2.61631	0.56534	4.62783	0.00000
dummy4	/dummy3	3.22516	0.63144	5.10761	0.00000
dummy4	/dummy4	4.18454	0.82442	5.07574	0.00000
dummy5	/dummy1	1.84738	0.34222	5.39825	0.00000
dummy5	/dummy2	2.84893	0.61934	4.59995	0.00000
dummy5	/dummy3	3.58710	0.68978	5.20034	0.00000
dummy5	/dummy4	4.69859	0.90930	5.16726	0.00000
dummy5	/dummy5	5.51998	1.04566	5.27894	0.00000
dummy6	/dummy1	2.17376	0.40991	5.30307	0.00000
dummy6	/dummy2	3.46369	0.77807	4.45162	0.00001
dummy6	/dummy3	4.38163	0.86682	5.05483	0.00000
dummy6	/dummy4	5.67539	1.12189	5.05879	0.00000
dummy6	/dummy5	6.58908	1.28273	5.13676	0.00000
dummy6	/dummy6	8.03920	1.60640	5.00448	0.00000



surveyhlm.OUT

LEVEL 2		TAU-HAT	STD.ERR.	Z-VALUE	PR > |Z|
dummy1	/dummy1	2.14267	0.18073	11.85561	0.00000
dummy2	/dummy1	1.45763	0.16138	9.03221	0.00000
dummy2	/dummy2	3.05585	0.31345	9.74921	0.00000
dummy3	/dummy1	1.93046	0.21077	9.15920	0.00000
dummy3	/dummy2	2.57313	0.29348	8.76777	0.00000
dummy3	/dummy3	4.32406	0.41703	10.36864	0.00000
dummy4	/dummy1	2.19516	0.22799	9.62828	0.00000
dummy4	/dummy2	3.11563	0.35917	8.67454	0.00000
dummy4	/dummy3	3.99632	0.41053	9.73441	0.00000
dummy4	/dummy4	5.92608	0.59607	9.94184	0.00000
dummy5	/dummy1	2.60165	0.27863	9.33742	0.00000
dummy5	/dummy2	3.88161	0.42861	9.05627	0.00000
dummy5	/dummy3	4.89265	0.48837	10.01828	0.00000
dummy5	/dummy4	6.06818	0.60663	10.00306	0.00000
dummy5	/dummy5	8.14693	0.75266	10.82416	0.00000
dummy6	/dummy1	2.88798	0.29610	9.75344	0.00000
dummy6	/dummy2	4.37308	0.53862	8.11907	0.00000
dummy6	/dummy3	5.73906	0.60068	9.55432	0.00000
dummy6	/dummy4	7.07370	0.71722	9.86268	0.00000
dummy6	/dummy5	8.61150	0.83782	10.27848	0.00000
dummy6	/dummy6	11.07427	1.13162	9.78620	0.00000

4.5 Evaluation

4.5.1 Introduction

A feature of many sampling surveys is that the probability of selection is unequal. This can be the result of stratified sampling, cluster sampling, subpopulation oversampling, designed unequal probability sampling, etc. If the unequal probability of selection is not incorporated in the analysis a substantial bias in the parameter estimates may arise. This bias is commonly known as the selection bias. If the probability of selection is known and incorporated in the analysis the selection bias can be eliminated.

In the next section we compare the performance of the methods implemented in the four statistical software packages LISREL, HLM, Mplus, and MLWiN. In all the tables to follow, the abbreviation MLevel is used to denote the multilevel module in LISREL.

4.5.2 Comparison of results using two-level simulated data

Asparouhov (2004) selected a linear growth model for continuous outcomes as the basis for a simulation study. This model can be estimated by all the different statistical packages for hierarchical linear modeling. An unbalanced design, consisting of 500 univariate observations that are clustered within 100 level two units, was used. Half of the level-2 units have four observations and the other half have six observations. The times of the observations are equally spaced starting at 0 and ending with 3 for the clusters with 4 observations and ending with 5 for the clusters with 6 observations. The linear growth model has random intercept and slope coefficients.

The observed variable Y_{ij} for level-2 unit i at time j is given by

$$Y_{ij} = b_{0i} + b_{1i} + \varepsilon_{ij}$$

where ε_{ij} is a zero-mean, normally distributed residual with variance σ^2. The random effects b_{0i} and b_{1i} have means β_0 and β_1, variances ϕ_{11} and ϕ_{22} respectively, and covariance ϕ_{21}.

The selection model is defined by the initial status in the growth model Y_{i0}, namely $P(b_{0i} = 1) = 1/(1+\exp\{-Y_{i0}\})$, *i.e.*, level-2 units with higher initial status have been oversampled. The analysis was replicated 500 times. An example of a few records for the first of the 500 data sets, in the form of a PSF, is shown below.

Simul.psf

	LEV2ID	Y	X	WT
1	10001.000	-0.484	0.000	2.623
2	10001.000	0.968	1.000	2.623
3	10001.000	2.165	2.000	2.623
4	10001.000	2.512	3.000	2.623
5	10002.000	0.256	0.000	1.774
6	10002.000	1.617	1.000	1.774
7	10002.000	0.555	2.000	1.774
8	10002.000	2.479	3.000	1.774
9	10002.000	4.189	4.000	1.774
10	10002.000	1.656	5.000	1.774

Table 4.5 shows the bias in the parameter estimates as well as the coverage rates for the 95% confidence intervals computed by LISREL 8.71, HLM 6, Mplus 3 and MLWiN 2. Note that this table contains updated HLM results when compared to Asparouhov (2004), where the previous version of HLM was used. In addition, results obtained with LISREL 8.7 have been added.

Table 4.5: Bias and Coverage in LISREL, HLM, MLWiN and HLM

Parameter	True Value	Bias				Coverage			
		LISREL	HLM	MLWiN	Mplus	LISREL	HLM	MLWiN	Mplus
β_0	0.5	0.019	0.016	0.017	0.017	0.906	0.908	0.782	0.908
β_1	0.1	0.001	0.03	0.002	0.002	0.948	0.938	0.888	0.942
ϕ_{11}	1	-0.029	-0.012	-0.024	-0.024	0.840	-	0.758	0.848
ϕ_{22}	0.2	-0.006	-0.001	-0.006	-0.006	0.878	-	0.848	0.902
ϕ_{21}	0.3	-0.005	-0.008	-0.005	-0.006	0.938	-	0.846	0.940
σ^2	1	-0.005	-0.012	-0.008	-0.008	0.946	-	0.878	0.910

The bias shown in subsequent tables is the difference between the mean of the estimated parameters over the 500 simulated data sets and the true value for that parameter as used in the actual simulation. For example, the first value for LISREL in the body of Table 4.5, *i.e.* 0.019, indicates that the average of the estimates of β_0 with this program was 0.5 + 0.019 = 0.519.

The coverage reported was calculated by determining the lower and upper bounds of a 95% confidence interval for each of the parameters for each of the simulated data sets. If the true value for the parameter fell within the confidence interval, an indicator variable was assigned a value of 1; if not, the indicator variable was coded 0. The mean value of the indicator variable over all 500 data sets, expressed as a percentage, is the coverage as reported in the tables to follow and indicates the percentage of data sets where the confidence interval based on the estimates obtained included the simulated or “true” value. The SAS code for calculation of coverage for the intercept is given below.

```
title 'coverage of intercept';
*Upper and lower limits of interval;
upper=intcept+1.96*serror;
lower=intcept-1.96*serror;
*Determine inclusion of true value;
if lower<=0.5<=upper then include=1;
else include=0;
proc means;
var include;
```

The biases produced by LISREL, HLM, Mplus and MLWiN are virtually identical. While the difference in the point estimation between the three methods is very small, the difference in the variance estimation (where available) is not.

Table 4.6 shows the effect of ignoring the design weight, as computed with LISREL 8.7. There is large bias present in the estimation of the intercept coefficient (β_0) and the variance of the level-2 intercept error term (ϕ_{11}) when weights are omitted. This conclusion is substantiated by the low coverage for these parameters.

Table 4.6 Bias and Coverage in LISREL without inclusion of a weight variable

Parameter	True Value	Without weights		With weights	
		Bias	Coverage	Bias	Coverage
β_0	0.5	-0.482	0.60	0.019	0.906
β_1	0.1	0.031	0.906	0.001	0.948
ϕ_{11}	-1.0	-0.312	0.592	-0.029	0.840
ϕ_{22}	0.2	-0.002	0.932	-0.006	0.878
ϕ_{21}	0.3	-0.020	0.922	-0.005	0.938
σ^2	1	-0.005	0.936	-0.005	0.946

4.5.3 Comparison of results using three-level simulated data

In this section we discuss the results of a simulation study for the evaluation of a 3-level model with level-2 and level-3 weights. Five hundred datasets were simulated according to the following hypothetical model

$$Score_{ijk} = \beta_0 + \beta_1 * Time + \gamma_1 * Lang1 + \gamma_2 * Lang2$$
$$+ v_{i0} + Time * v_{i1} + u_{ij0} + Time * u_{ij1} + e_{ijk}$$

where i denotes institution i, (i = 1, 2, …, 100), ij patient j (j =1, 2, …, 10) in institution i and ijk the k-th measurement (k = 1, 2, …, 6) on patient j in institution i. The outcome variable Score denotes a patient's measurement on some test of interest, TIME the time of measurement, and Lang1 and Lang2 are indicator variables indicating a patient's first or home language. The data were simulated under the assumption that

$$\begin{pmatrix}\beta_0\\ \beta_1\end{pmatrix}=\begin{pmatrix}1.0\\ 0.5\end{pmatrix},\quad \begin{pmatrix}\gamma_1\\ \gamma_2\end{pmatrix}=\begin{pmatrix}0.5\\ -1.0\end{pmatrix}$$

$$\phi_2 = Cov(u_{ij0}, u_{ij1}) = \begin{pmatrix}1 & \\ 0.3 & 0.2\end{pmatrix}$$

$$\phi_3 = Cov(v_{i0}, v_{i1}) = \begin{pmatrix}1 & \\ 0.3 & 0.2\end{pmatrix}$$

and

$$\sigma^2 = Var(e_{ijk}) = 1.0.$$

Note that the data were simulated in such a way that odd-numbered patients have six score measurements at time points 0, 1, 2, 3, 4, 5. The even-numbered patients have only four score measurements.

Level-3 weights

To incorporate design weights, the simulated initial scores were standardized to a normal (0,1) distribution and an equal number of "institutions" were subsequently drawn from each of the 10 score intervals ($(-\infty,-1)$, $(-1,-0.7)$, …, $(2.3,\infty)$) as shown in the table below. It can easily be verified that for a standardized normal variable z, $P(0.30 \le z \le 1.00) = 22.34\%$. In the simulation study, patients (cases) were selected from 10 institutions if their standardized scores fell within the interval (0.30, 1.00). To correct for this undersampling, a weight of 22.34/10.0 = 2.234 was assigned to each of those institutions. In a similar fashion, 10 institutions were selected according to the remaining nine score intervals as shown below.

Interval	Lower	Upper	% Expected	% Selected	Weight3
1	-Inf	-1.00	15.87	10.00	1.587
2	-1.00	-0.70	8.33	10.00	0.833
3	-0.70	-0.20	17.88	10.00	1.788
4	-0.20	0.00	7.93	10.00	0.793
5	0.00	0.30	11.79	10.00	1.179
6	0.30	1.00	22.34	10.00	2.234
7	1.00	1.30	6.19	10.00	0.619
8	1.30	1.80	6.09	10.00	0.609
9	1.80	2.30	2.52	10.00	0.252
10	2.30	Inf	1.07	10.00	0.107

Level-2 weights

In order to incorporate level-2 weights, it was further assumed that the actual percentages of patients in each of three ethnic groups are 70%, 20% and 10%. However, in each institution four patients were drawn from the first ethnic groups, and three from each of the second and third ethnic groups. To compensate for this unequal probability of selection, level-2 ("patient") weights were assigned as follows:

- Four from ethnic group 1 with Weight2 = 7.0/4.0
- Three from ethnic group 2 with Weight2 = 2.0/3.0
- Three from ethnic group 3 with Weight2 = 1.0/3.0

The first 10 records of the dataset in **surveyhlm.psf** are shown below.

surveyhlm.psf

	Institut	Patient	Score	Time	Lang1	Lang2	WT3	WT2
1	1.00	1.00	-1.84	0.00	0.00	0.00	1.59	1.75
2	1.00	1.00	-0.89	1.00	0.00	0.00	1.59	1.75
3	1.00	1.00	-1.21	2.00	0.00	0.00	1.59	1.75
4	1.00	1.00	-3.24	3.00	0.00	0.00	1.59	1.75
5	1.00	1.00	-1.16	4.00	0.00	0.00	1.59	1.75
6	1.00	1.00	-1.30	5.00	0.00	0.00	1.59	1.75
7	1.00	2.00	-0.07	0.00	0.00	0.00	1.59	1.75
8	1.00	2.00	2.99	1.00	0.00	0.00	1.59	1.75
9	1.00	2.00	0.92	2.00	0.00	0.00	1.59	1.75
	1.00	2.00	3.63	3.00	0.00	0.00	1.59	1.75

The model was fitted to 500 simulated data sets using HLM 6.0 (Bryk & Raudenbush, 2004) and LISREL. Table 4.7 shows the bias and coverage for estimates obtained with weighted analyses using LISREL and HLM. Results are very similar. HLM 6.0 does not provide estimates of the variance-covariance components in the case of weighted models. Due to this, coverage for the HLM results could not be calculated.

Table 4.8 shows the effect of ignoring the design weight, as computed with LISREL 8.7. For the unweighted analyses it was found that the estimates of the fixed parameters (β_0, β_1, γ_1 and γ_2) were strongly biased as is also reflected by the low coverage (0.042 in the case of the intercept coefficient). As was the case for the similar comparison shown in Table 4.6, both bias and coverage for the weighted model yield closer approximations to the theoretical expected values for bias and coverage (0 and 0.95).

Table 4.7 Bias and Coverage for simulated three-level data (weighted analyses)

Parameter	True Value	LISREL		HLM	
		Bias	Coverage	Bias	Coverage
β_0	1.0	-0.002	0.986	-0.002	0.986
β_1	0.5	0.001	0.948	-0.004	0.948
γ_1	0.5	-0.004	0.950	0.001	0.950
γ_2	-1.0	-0.004	0.936	-0.004	0.936
σ^2	1.0	0.001	1.000	0.010	-
$\phi_{(2)11}$	1.0	-0.072	0.856	-0.075	-
$\phi_{(2)22}$	0.2	-0.003	0.970	-0.000	-
$\phi_{(2)21}$	0.3	-0.011	0.906	-0.013	-
$\phi_{(3)11}$	1.0	-0.049	0.930	0.049	-
$\phi_{(3)22}$	0.2	-0.002	0.922	-0.003	-
$\phi_{(3)21}$	0.3	0.008	0.922	0.008	-

Table 4.8 Bias and Coverage for simulated three-level data (unweighted analyses)

Parameter	True Value	Bias	Coverage
β_0	1.0	0.418	0.042
β_1	0.5	0.090	0.548
γ_1	0.5	-0.115	0.756
γ_2	-1.0	-0.122	0.764
σ^2	1.0	0.004	0.920
$\phi_{(2)11}$	1.0	0.064	0.908
$\phi_{(2)22}$	0.2	0.000	0.966
$\phi_{(2)21}$	0.3	0.006	0.958
$\phi_{(3)11}$	1.0	0.256	0.850
$\phi_{(3)22}$	0.2	0.017	0.954
$\phi_{(3)21}$	0.3	0.070	0.914

4.5.4 Comparison of results using a 3-level model for the MEPS data

The model fitted in Section 4.4.1 using the multilevel module (MLevel) of LISREL was also fitted using HLM 6, MLWiN 2 and the SurveyGLIM (GLIM) module of LISREL 8.7. Table 4.9 below contains estimates obtained with these four procedures for both weighted and unweighted models. Standard error estimates, where available, are given below the estimates in parentheses. Simplified two-level models using the same data and a wider array of statistical software are given in the next section.

Results of the GLIM analyses are not directly comparable to those obtained using the multilevel analysis programs HLM, MLevel and MLWiN, but are presented here to demonstrate the effect of the different modeling assumptions. The standard errors reported for the SurveyGLIM module were obtained under the assumption of stratification and clustering, using a Taylor linearization approach to the asymptotic covariance matrix (see Section 3.6). In the multilevel programs, the stratum and cluster variables define the hierarchical structure by serving as level-3 and level-2 identifiers, and it is assumed that the intercept coefficients vary randomly across the level-3 and level-2 units.

Table 4.9 Comparison of results from 4 procedures for model fitted to MEPS data

Coefficient	Weighted				Unweighted			
	HLM	**MLevel**	**MLWiN**	**GLIM**	**HLM**	**MLevel**	**MLWiN**	**GLIM**
intercept	4.360 (0.122)	4.391 (0.114)	4.336 (0.108)	4.277 (0.125)	4.458 (0.115)	4.458 (0.084)	4.459 (0.084)	4.282 (0.025)
race	0.944 (0.088)	0.943 (0.086)	0.939 (0.082)	0.880 (0.100)	0.684 (0.096)	0.684 (0.050)	0.684 (0.050)	0.623 (0.017)
sex	0.904 (0.038)	0.911 (0.036)	0.920 (0.039)	0.932 (0.041)	0.931 (0.037)	0.931 (0.036)	0.931 (0.036)	0.945 (0.013)
inscov	-0.616 (0.081)	-0.651 (0.076)	-0.695 (0.074)	-0.630 (0.086)	-0.618 (0.084)	-0.618 (0.046)	-0.618 (0.046)	-0.733 (0.016)
rpovc991	0.363 (0.118)	0.358 (0.114)	0.432 (0.103)	0.439 (0.109)	0.493 (0.095)	0.493 (0.065)	0.493 (0.065)	0.668 (0.023)
rpovc992	-0.122 (0.110)	-0.138 (0.104)	-0.142 (0.106)	-0.114 (0.107)	-0.154 (0.097)	-0.154 (0.067)	-0.154 (0.067)	-0.128 (0.024)
rpovc993	0.088 (0.111)	0.070 (0.117)	0.119 (0.097)	0.130 (0.116)	0.101 (0.096)	0.101 (0.062)	0.101 (0.062)	0.205 (0.022)
rpovc994	-0.318 (0.152)	-0.329 (0.140)	-0.343 (0.150)	-0.263 (0.141)	-0.346 (0.109)	-0.346 (0.090)	-0.346 (0.090)	-0.336 (0.032)
variance (level-1)	6.682	7.006 (0.196)	7.233 (0.166)	N/A	7.463 (0.069)	7.463 (0.069)	7.463 (0.069)	N/A
variance (level-2)	0.190	0.177 (0.037)	0.200 (0.046)	N/A	0.175 (0.028)	0.174 (0.028)	0.174 (0.028)	N/A
variance (level-3)	0.079	0.073 (0.029)	0.101 (0.048)	N/A	0.145 (0.037)	0.146 (0.0365)	0.146 (0.036)	N/A
deviance	112067	118256	114698		114663	114663	114663	
parameters	11	11	11		11	11	11	

Parameter estimates for the unweighted analyses with HLM, MLevel and MLWiN are identical. The HLM standard error estimates for the fixed effects are generally larger than those reported for MLevel and MLWiN. The reason for this is that the robust standard error estimates produced by HLM were reported. By including the commands

```
WEIGHT2 = intcept;
WEIGHT3 = intcept;
```

in the MLevel syntax file, results similar to those produced by HLM can be obtained with the MLevel module (see Section 4.6.4 for a further discussion of this topic). The parameter and standard error estimates for the HLM, MLevel and MLWiN procedures are very similar. Note that HLM 6.0 does not produce standard errors for the variance components.

Turning to the GLIM results, we note relatively large differences in parameter and standard error estimates for both the weighted and unweighted models. More research may be required to provide users with guidelines if a choice has to be made between fitting a multilevel or a generalized linear model to complex survey data.

4.5.5 Comparison of results using a 2-level model for the MEPS data

In order to expand the comparison of results for weighted models using all the software packages at our disposal, we fitted a series of two-level models to the MEPS data. While this implies ignoring the survey sample design to some extent, doing so was necessary in order to obtain results for SAS PROC MIXED and Mplus. In the case of PROC MIXED, fitting 3-level models is computationally intensive and thus not an option when a large number of models is to be fitted, while Mplus cannot presently accommodate level-3 models.

In the first set of models, it was assumed that respondents were nested within the 143 strata only, and no distinction was made in terms of the PSU they were drawn from. Three models were introduced, each using a different subset of the predictors used in Section 4.4.1. For each of the models, results for both weighted and unweighted analyses are given in Tables 4.10, 4.11 and 4.12.

An inspection of Tables 4.10 to 4.12 shows that the GLIM estimates and standard errors differ from those obtained using the multilevel procedures. This result is to be expected, since the multilevel approach allows for all or a subset of the regression coefficients to vary randomly over the different levels of the hierarchical structure, while the GLIM approach assumes fixed regression coefficients and uses stratification and clustering variables to produce appropriate standard errors.

Table 4.10 Comparison of results for first model fitted to MEPS data

Model 1: Results for unweighted analyses						
Coefficients	**MLevel**	**HLM**	**SAS**	**MLWiN**	**GLIM**	**Mplus**
intercept	4.652 (0.066)	4.652 (0.092)	4.652 (0.066)	4.652 (0.066)	4.571 (0.078)	4.654 (0.093)
race	0.715 (0.048)	0.715 (0.097)	0.715 (0.048)	0.725 (0.048)	0.685 (0.117)	0.713 (0.097)
sex	0.919 (0.036)	0.919 (0.036)	0.919 (0.036)	0.929 (0.036)	0.924 (0.039)	0.920 (0.036)
inscov	-0.838 (0.040)	-0.838 (0.081)	-0.838 (0.040)	-0.838 (0.040)	-1.015 (0.124)	-0.840 (0.081)
variance (level 1)	7.632 (0.071)	7.633	7.632	7.632 (0.071)	N/A	7.634 (0.181)
variance (level 2)	0.221 (0.036)	0.224	0.222	0.333 (0.036)	N/A	0.223 (0.036)
Model 1: Results for weighted analyses						
Coefficients	**MLevel**	**HLM**	**SAS**	**MLWiN**	**GLIM**	**Mplus**
intercept	4.527 (0.092)	4.528 (0.093)	4.514 (0.062)	4.541 (0.091)	4.470 (0.091)	4.527 (0.093)
race	0.970 (0.082)	0.970 (0.082)	0.966 (0.048)	0.920 (0.084)	0.985 (0.086)	0.970 (0.082)
sex	0.902 (0.036)	0.902 (0.036)	0.902 (0.035)	0.906 (0.040)	0.901 (0.037)	0.902 (0.036)
inscov	-0.822 (0.078)	-0.822 (0.078)	-0.825 (0.041)	-0.832 (0.095)	-0.899 (0.099)	-0.824 (0.078)
variance (level 1)	7.157 (0.243)	6.337	7.158	7.357 (0.190)	N/A	7.161 (0.168)
variance (level 2)	0.139 (0.019)	0.148	0.161	0.172 (0.027)	N/A	0.138 (0.020)

Table 4.11 Comparison of results for second model fitted to MEPS data

Model 2: Results for unweighted analyses						
Coefficients	**MLevel**	**HLM**	**SAS**	**MLWiN**	**GLIM**	**Mplus**
intercept	4.335 (0.067)	4.335 (0.085)	4.335 (0.067)	4.335 (0.067)	4.160 (0.089)	4.338 (0.086)
race	0.821 (0.048)	0.821 (0.101)	0.821 (0.048)	0.821 (0.048)	0.810 (0.122)	0.819 (0.101)
sex	0.904 (0.036)	0.904 (0.035)	0.904 (0.036)	0.904 (0.036)	0.906 (0.038)	0.905 (0.035)
variance (level 1)	7.765 (0.072)	7.765	7.765	7.765 (0.072)	N/A	7.768 (0.197)
variance (level 2)	0.275 (0.042)	0.224	0.274	0.275 (0.042)	N/A	0.276 (0.048)

Table 4.11 (continued) Comparison of results for second model fitted to MEPS data

Model 2: Results for weighted analyses						
Coefficients	**MLevel**	**HLM**	**SAS**	**MLWiN**	**GLIM**	**Mplus**
intercept	4.210 (0.085)	4.211 (0.085)	4.184 (0.062)	4.220 (0.082)	4.119 (0.091)	4.209 (0.085)
race	1.108 (0.080)	1.108 (0.080)	1.105 (0.048)	1.044 (0.086)	1.134 (0.084)	1.108 (0.081)
sex	0.894 (0.037)	0.894 (0.037)	0.894 (0.035)	0.896 (0.040)	0.892 (0.037)	0.894 (0.037)
variance (level 1)	7.275 (0.240)	6.442	7.276	7.482 (0.207)	N/A	7.279 (0.177)
variance (level 2)	0.161 (0.023)	0.148	0.191	0.205 (0.034)	N/A	0.160 (0.024)

Table 4.12 Comparison of results for third model fitted to MEPS data

Model 3: Results for unweighted analyses						
Coefficients	**MLevel**	**HLM**	**SAS**	**MLWiN**	**GLIM**	**Mplus**
intercept	5.143 (0.064)	5.144 (0.095)	5.144 (0.064)	5.144 (0.063)	5.066 (0.078)	5.148 (0.096)
race	0.692 (0.049)	0.692 (0.095)	0.692 (0.049)	0.692 (0.049)	0.660 (0.116)	0.689 (0.095)
inscov	-0.817 (0.041)	-0.817 (0.082)	-0.817 (0.041)	-0.817 (0.041)	-0.995 (0.124)	-0.819 (0.081)
variance (level 1)	7.843 (0.073)	7.844	7.843	7.843 (0.072)	N/A	7.843 (0.184)
variance (level 2)	0.221 (0.036)	0.223	0.221	0.221 (0.036)	N/A	0.221 (0.036)

Model 3: Results for weighted analyses						
Coefficients	**MLevel**	**HLM**	**SAS**	**MLWiN**	**GLIM**	**Mplus**
intercept	5.000 (0.089)	5.001 (0.090)	4.987 (0.060)	5.012 (0.090)	4.945 (0.088)	4.999 (0.090)
race	0.952 (0.081)	0.952 (0.081)	0.948 (0.048)	0.905 (0.083)	0.967 (0.086)	0.952 (0.081)
inscov	-0.810 (0.079)	-0.810 (0.079)	-0.813 (0.042)	-0.818 (0.096)	-0.887 (0.100)	-0.812 (0.079)
variance (level 1)	7.361 (0.249)	6.517	7.361	7.562 (0.192)	N/A	7.365 (0.169)
variance (level 2)	0.137 (0.019)	0.146	0.160	0.172 (0.027)	N/A	0.136 (0.020)

For all the weighted analyses, a comparison of the multilevel (MLevel) and Mplus results show that the estimated parameters and standard errors (given in parentheses) are almost identical, with the exception of the standard error estimate for the level-1 variance component.

A comparison of the results for the unweighted analyses reveals differences in standard error estimates. In this case, the MLevel, SAS, and MLWiN standard error estimates are in close agreement, while those produced by HLM and Mplus are the same. In Section 4.6.4 it is shown that robust standard error estimates can be obtained in the unweighted case if the command WEIGHT1 = intcept is included in the syntax file. In doing so, the LISREL MLevel method yields the standard errors reported in the Mplus column.

4.6 Theory

4.6.1 Introduction

In Section 4.6.2, we outline a general procedure for the implementation of weights in level-2 and level-3 models. A more rigorous theoretical treatment of these results are presented in Section 4.6.3. In Section 4.6.4 we provide some results for standard error estimation and fit statistics.

4.6.2 A general weighting procedure

Under the assumption that the sampling weights at a specific level are independent of the random effects at that level, Pfeffermann *et. al.* (1997) adopted the following procedure. Consider the case of a 2 level model. Denote by w_i the weight attached to the *i*-th level-2 unit and by $w_{j|i}$ the weight attached to the *j*-th level 1 unit within the *i*-th level-2 unit such that

$$\sum_j w_{j|i} = n_i, \sum_i w_i = I$$

where *I* is the total number of level 2 units and $N = \sum_i n_i$ the total number of level-1 units. That is, the lower level weights within each immediate higher level unit are scaled to have a mean of unity, and likewise for higher levels. For each level 1 unit we now form the final, or composite, weight

$$w_{ji} = Nw_{j|i}w_i \Big/ \sum_j \sum_i w_{j|i}w_i = Nw_{j|i}w_i \Big/ \sum_i n_i w_i \,.$$

Denote by $\mathbf{z}_u$ and $\mathbf{z}_e$ respectively the sets of explanatory variables defining the level 2 and level 1 random coefficients and form

$$\mathbf{z}_u^* = \mathbf{W}_i \mathbf{z}_u, \mathbf{W}_i = Diag\left\{w_i^{-0.5}\right\}$$
$$\mathbf{z}_e^* = \mathbf{W}_{ji} \mathbf{z}_e, \mathbf{W}_{ji} = Diag\left\{w_{ji}^{-0.5}\right\}.$$

We now carry out a standard estimation but using $\mathbf{z}_u^*$ and $\mathbf{z}_e^*$ as the random coefficient explanatory variables. For a 3 level model, with an obvious extension to notation, we have the following

$$\sum_j w_{j|ik} = n_{ik}, \sum_i w_{i|k} = I_k, \sum_k w_k = K, N = \sum_i \sum_k n_{ik}, I = \sum_k I_k$$
$$w_{jik} = Nw_{j|ik} w_{i|k} w_k \Big/ \sum_j \sum_i \sum_k w_{j|ik} w_{i|k} w_k, w_{ik} = Iw_{i|k} w_k \Big/ \sum_i \sum_k w_{i|k} w_k.$$

Goldstein (1995) also pointed out that in survey work analysts often have access only to the final level-1 weights w_{ji}. In this case, say for a 2-level model, we can obtain the w_i by computing $w_i^{'} = W_i I \Big/ \sum_i W_i, W_i = \left(\sum_j w_{ji} \right) \Big/ n_i$. For a 3-level model the procedure is carried out for each level-3 unit and the resulting $w_{ik}^{'}$ are transformed analogously.

4.6.3 Weights in multilevel models

Let

$$\mathbf{y}_i = \mathbf{X}_{(f)i}\boldsymbol{\beta} + \mathbf{X}_{(2)i}\mathbf{u}_i + \sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\mathbf{e}_{ij}, \quad i = 1, 2, ..., I$$

with $\mathbf{y}_i : (n_i \times 1)$, $\mathbf{X}_{(f)i} : n \times p$, $\mathbf{X}_{(2)i} : n_i \times q$, and $\mathbf{Z}_{(1)ij} : n_i \times r$. It is further assumed that $\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_I$ are independently and identically distributed (i.i.d) with $E(\mathbf{u}_i) = \mathbf{0}$, $Cov(\mathbf{u}_i) = \boldsymbol{\Phi}_{(2)}$. Also, $\mathbf{e}_{11}, ..., \mathbf{e}_{1n_1}, \mathbf{e}_{21}, ..., \mathbf{e}_{In_I}$ are i.i.d. with $E(\mathbf{e}_{ij}) = \mathbf{0}$, $Cov(\mathbf{e}_{ij}) = \boldsymbol{\Phi}_{(1)}$. Note further that

$$Z_{(1)ij} = \begin{bmatrix} \mathbf{0}^{'} \\ \mathbf{0}^{'} \\ \mathbf{x}_{(1)ij}^{'} \\ \mathbf{0}^{'} \\ \vdots \\ \mathbf{0}^{'} \end{bmatrix}.$$

Example (r=1):

Suppose that $\mathbf{x}_{(1)ij}^{'} = 1, i = 1, 2, ..., I, j = 1, 2, ..., n_i$. In this case, $Cov\left[\sum_{j=1}^{n_i} \mathbf{Z}_{(1)ij}\mathbf{e}_{ij} \right] = \sigma^2 \mathbf{I}_{n_i}$, where $\sigma^2 = \Phi_{(1)}$, a scalar. From the distributional assumptions given above, it follows that

$$E(\mathbf{y}_i) = \mathbf{X}_{(f)i}\boldsymbol{\beta}, \ Cov(\mathbf{y}_i, \mathbf{y}_i^{'}) = \boldsymbol{\Sigma}_i$$

where

$$\mathbf{\Sigma}_i = \mathbf{X}_{(2)i}\mathbf{\Phi}_{(2)}\mathbf{X}_{(2)i}^{'} + \sum_{j=1}^{n_i}\mathbf{Z}_{(1)ij}\mathbf{\Phi}_{(1)}\mathbf{Z}_{(1)ij}^{'}.$$

Consider the case where $r = 1$, then $\Phi_{(1)} = \sigma_e^2$ and $\sum_{j=1}^{n_i}\mathbf{Z}_{(1)ij}\mathbf{\Phi}_{(1)}\mathbf{Z}_{(1)ij}^{'} = \mathbf{D}_i\sigma^2$.

If $\mathbf{Z}_{(1)ij} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ x_{(1)ij} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, then $\mathbf{Z}_{(1)ij}\mathbf{Z}_{(1)ij}^{'} = \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & x_{(1)ij}^2 & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix}$,

and hence $\sum_{j=1}^{n_i}\mathbf{Z}_{(1)ij}\sigma^2\mathbf{Z}_{(1)ij}^{'} = \sigma^2 diag(x_{(1)i1}^2,\ldots,x_{(1)in_i}^2) = \mathbf{D}_i\sigma^2$.

Let $\mathbf{V}_i$ be a provisional estimate of $\mathbf{\Sigma}_i$, then

$$\boldsymbol{\beta} \mid \mathbf{V}_i = \left[\sum_{i=1}^{I}\mathbf{X}_{(f)i}^{'}\mathbf{V}_i^{-1}\mathbf{X}_{(f)i}\right]^{-1}\left[\sum_{i=1}^{I}\mathbf{X}_{(f)i}^{'}\mathbf{V}_i^{-1}\mathbf{y}_i\right],$$

where

$$\begin{aligned}\mathbf{V}_i &= \mathbf{X}_{(2)i}\hat{\mathbf{\Phi}}_{(2)}\mathbf{X}_{(2)i}^{'} + \mathbf{D}_i\hat{\sigma}^2 \\ &= \hat{\sigma}^2\left(\mathbf{X}_{(2)i}\frac{\hat{\mathbf{\Phi}}_{(2)}}{\hat{\sigma}^2}\mathbf{X}_{(2)i}^{'} + \mathbf{D}_i\right).\end{aligned}$$

Using a well-known result for matrix inversion,

$$\left[\mathbf{B\Omega B}^{'} + \mathbf{\Lambda}\right]^{-1} = \mathbf{\Lambda}^{-1} - \mathbf{\Lambda}^{-1}\mathbf{B}\left[\mathbf{\Omega}^{-1} + \mathbf{B}^{'}\mathbf{\Lambda}^{-1}\mathbf{B}\right]^{-1}\mathbf{B}^{'}\mathbf{\Lambda}^{-1},$$

it follows that

$$\mathbf{V}_i^{-1} = \left(\hat{\sigma}^2\right)^{-1}\left\{\mathbf{D}_i^{-1} - \mathbf{D}_i^{-1}\mathbf{X}_{(2)i}\left(\hat{\sigma}^2\mathbf{\Phi}_{(2)}^{-1} + \mathbf{X}_{(2)i}^{'}\mathbf{D}_i^{-1}\mathbf{X}_{(2)i}\right)^{-1}\mathbf{X}_{(2)i}^{'}\mathbf{D}_i^{-1}\right\}.$$

Hence

$$\mathbf{X}_{(f)i}^{'}\mathbf{V}_i^{-1}\mathbf{X}_{(f)i} = \left(\hat{\sigma}^2\right)^{-1}\left\{\mathbf{T}_{1i} - \mathbf{T}_{2i}\left[\hat{\sigma}^2\hat{\mathbf{\Phi}}_{(2)}^{-1} + \mathbf{T}_{3i}\right]^{-1}\mathbf{T}_{2i}^{'}\right\},$$

where

$$\mathbf{T}_{1i} = \mathbf{X}_{(f)i}^{'}\mathbf{D}_i^{-1}\mathbf{X}_{(f)i} = \sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}\mathbf{x}_{(f)ij}^{'} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_{1i} = \sum_{j=1}^{n_i} w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}_{(f)ij}^{'} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_1 = \sum_{i=1}^{I} w_i \sum_{j=1}^{n_i} w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}_{(f)ij}^{'} / x_{(1)ij}^2.$$

Since $w_{ij} = w_i \cdot w_{j|i}$, it follows that

$$\hat{\mathbf{T}}_i = \sum_{i=1}^{I}\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}\mathbf{x}_{(f)ij}^{'} / x_{(1)ij}^{*2},$$

where $x_{(1)ij}^{*} = w_{ij}^{-1/2}x_{(1)ij}$.

$$\mathbf{T}_{2i} = \sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}\mathbf{x}_{(2)ij}^{'} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_{2i} = \sum_{j=1}^{n_i} w_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}_{(2)ij}^{'} / x_{(1)ij}^2$$

$$\mathbf{T}_{3i} = \sum_{j=1}^{n_i}\mathbf{x}_{(2)ij}\mathbf{x}_{(2)ij}^{'} / x_{(1)ij}^2$$

$$\hat{\mathbf{T}}_{3i} = \sum_{j=1}^{n_i}(w_i \cdot w_i^{-1})w_{j|i}\mathbf{x}_{(2)ij}\mathbf{x}_{(2)ij}^{'} / x_{(1)ij}^2$$

$$= \sum_{j=1}^{n_i} w_i^{-1}w_{ij}\mathbf{x}_{(2)ij}\mathbf{x}_{(2)ij}^{'} / x_{(1)ij}^2$$

$$= \sum_{j=1}^{n_i}\mathbf{x}_{(2)ij}^{*}\mathbf{x}_{(2)ij}^{*'} / x_{(1)ij}^{*2}$$

where $\mathbf{x}^*_{(2)ij} = w_i^{-1/2}\mathbf{x}_{(2)ij}$.

Let

$$\hat{\mathbf{C}}_{(2)i} = \left[\hat{\sigma}^2\hat{\mathbf{\Phi}}^{-1}_{(2)} + \hat{\mathbf{T}}_{3i}\right]^{-1}.$$

then $\sum_{i=1}^{I}\mathbf{T}_{2i}\mathbf{C}_{(2)i}\mathbf{T}^{'}_{2i}$ is estimated by

$$\sum_{i=1}^{I} w_i\hat{\mathbf{T}}_{2i}\hat{\mathbf{C}}_{(2)i}\hat{\mathbf{T}}^{'}_{2i} = \sum_{i=1}^{I} w_i^{1/2}\left[\sum_{j=1}^{n_i} w_i^{-1/2}w_i^{-1/2}w_iw_{j|i}\mathbf{x}_{(f)ij}\mathbf{x}^{'}_{(2)ij}/x^2_{(1)ij}\right]\times$$
$$\hat{\mathbf{C}}_{(2)i}w_i^{1/2}\left[\sum_{j=1}^{n_i} w_i^{-1/2}w_i^{-1/2}w_iw_{j|i}\mathbf{x}_{(2)ij}\mathbf{x}^{'}_{(f)ij}/x^2_{(1)ij}\right].$$

Since $w_iw_{j|i} = w_{ij}$, it follows that $\sum_{i=1}^{I}\mathbf{T}_{2i}\mathbf{C}_{(2)i}\mathbf{T}^{'}_{2i}$ is estimated by

$$\sum_{i=1}^{I}\left[\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}\mathbf{x}^{*'}_{(2)ij}/x^{*2}_{(1)ij}\right]\hat{\mathbf{C}}_{(2)i}\left[\sum_{j=1}^{n_i}\mathbf{x}^*_{(2)ij}\mathbf{x}^{'}_{(f)ij}/x^{*2}_{(1)ij}\right].$$

Similarly

$$\mathbf{X}^{'}_{(f)i}\mathbf{V}_i^{-1}\mathbf{y}_i = \left(\hat{\sigma}^2\right)^{-1}\left[\mathbf{X}^{'}_{(f)i}\mathbf{D}_i^{-1}\mathbf{y}_i - \mathbf{X}^{'}_{(f)i}\mathbf{D}_i^{-1}\mathbf{X}_{(2)i}\mathbf{C}_{(2)i}\mathbf{X}^{'}_{(2)i}\mathbf{D}_i^{-1}\mathbf{y}_i\right]$$
$$= \left(\hat{\sigma}^2\right)^{-1}\left[\mathbf{t}_{4i} - \mathbf{T}_{2i}\mathbf{C}_{(2)i}\mathbf{t}_{5i}\right]$$

where

$$\mathbf{t}_{4i} = \mathbf{X}^{'}_{(f)i}\mathbf{D}_i^{-1}\mathbf{y}_i = \sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}y_{ij}/x^2_{(1)ij},$$

$$\hat{\mathbf{t}}_{4i} = \sum_{j=1}^{n_i} w_i^{-1}w_iw_{i|j}\mathbf{x}_{(f)ij}y_{ij}/x^2_{(1)ij} = w_i^{-1}\sum_{j=1}^{n_i}\mathbf{x}_{(f)ij}y_{ij}/x^{*2}_{(1)ij}.$$

$$\mathbf{t}_{5i} = \mathbf{X}^{'}_{(2)i}\mathbf{D}_i^{-1}\mathbf{y}_i = \sum_{j=1}^{n_i}\mathbf{x}_{(2)ij}y_{ij}/x^2_{(1)ij}$$

$$\hat{\mathbf{t}}_{5i} = w_i^{-1/2}\sum_{j=1}^{n_i}\mathbf{x}^*_{(2)ij}y_{ij}/x^{*2}_{(1)ij}.$$

It can then be shown that $\sum_{i=1}^{I} \mathbf{X}_{(f)i}^{'} \mathbf{V}_i^{-1} \mathbf{y}_i$ is estimated by $\left(\hat{\sigma}^2\right)^{-1} \left[\hat{\mathbf{t}}_{4i} - \sum_{i=1}^{I} \hat{\mathbf{q}}_i\right]$ where

$$\hat{\mathbf{q}}_i = \left[\sum_{j=1}^{n_i} \mathbf{x}_{(f)ij} \mathbf{x}_{(2)ij}^{*'} / x_{(1)ij}^{*2}\right] \cdot \hat{\mathbf{C}}_{(2)i} \cdot \left[\sum_{j=1}^{n_i} \mathbf{x}_{(2)ij}^{*} y_{ij} / x_{(1)ij}^{*2}\right] \text{ and } \hat{\mathbf{t}}_4 = \sum_{i=1}^{I} \widehat{w_i \mathbf{t}_{4i} = \sum_{i=1}^{I}} \sum_{j=1}^{n_i} \mathbf{x}_{(f)ij} y_{ij} / x_{(1)ij}^{*2}.$$

4.6.4 Standard errors and fit statistics

The method used in LISREL to calculate standard error estimates for multilevel models depends on whether design weights are included in the analysis or not. We first consider the case where design weights are used.

Let $\hat{\boldsymbol{\gamma}}$ denote the vector of estimated parameters. In Section 2.8 it was shown that an approximate expression for the asymptotic covariance matrix of $\hat{\boldsymbol{\gamma}}$ is given by

$$Cov(\hat{\boldsymbol{\gamma}}) \approx \mathbf{I}_n^{-1}(\boldsymbol{\gamma}) \mathbf{G} \mathbf{I}_n^{-1}(\boldsymbol{\gamma})$$

where

$$E\left[\frac{\partial^2 \ln L}{\partial \boldsymbol{\gamma} \partial \boldsymbol{\gamma}^{'}}\right] = -\mathbf{I}_n(\boldsymbol{\gamma}).$$

As an estimate of $\mathbf{G}$ we use

$$\mathbf{G} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{g}_i \mathbf{g}_i^{'},$$

where $\mathbf{g}_i$ denotes the i-th contribution to the gradient vector $\mathbf{g} = \dfrac{\partial \ln L}{\partial \boldsymbol{\gamma}}$.

Standard error estimates of the unknown parameters are obtained by taking the square roots of the diagonal elements of $Cov\left(\hat{\boldsymbol{\gamma}}\right)$.

If no weighting variable is specified, it is assumed that $Cov\left(\hat{\boldsymbol{\gamma}}\right)$ equals the inverse of the information matrix, that is $Cov\left(\hat{\boldsymbol{\gamma}}\right) = \mathbf{I}_n^{-1}$. In the case where no weight is specified, so-called robust standard error estimates of the estimated parameters may be obtained by using the asymptotic covariance matrix for the weighted case. This is accomplished by adding the command WEIGHT1 = intcept; to the multilevel syntax file.

Likelihood ratio tests

Test of a null hypothesis against a restricted alternative hypothesis can be constructed, provided that two conditions are met. Firstly, the models under H_0 and H_1 should be estimable and secondly, the parameter space $\boldsymbol{\Omega}_0$ for H_0 must be a subset of the parameter space $\boldsymbol{\Omega}$ for H_1.

Use is made of the likelihood ratio test statistic

$$\lambda = \frac{L_0\left(\hat{\boldsymbol{\gamma}}_0\right)}{L_1\left(\hat{\boldsymbol{\gamma}}_1\right)}$$

where L_0 and L_1 denote the likelihood functions under H_0 and H_1 respectively. For a large N (see, for example, Anderson, 1984), $-2\ln\lambda = -2\ln L_0 - \left(-2\ln L_1\right)$ has an approximate $\chi^2_{(v)}$ distribution where the number of degrees of freedom v is the difference in the number of parameters estimated under H_0 and the number of parameters estimated under H_1. The statistic $-2\ln L$ is called the deviance.

Contrasts

Consider a clinical trial in which two types of drugs are administered to 400 obese adults. Adults are randomly assigned to four groups:

- Group 1, Drug A, low dosage (10 mg/day)
- Group 2, Drug A, high dosage (50 mg/day)
- Group 3, Drug B, low dosage (10 mg/day)
- Group 4, Drug B, high dosage (50 mg/day)

Let y_{ij} denote weight loss of subject *i* on occasion t_j, *i* = 1, 2, …, 400 and *j* = 1, 2, …, n_i, and let

$$\begin{aligned} y_{ij} = {} & \beta_1 AL + \beta_2 AH + \beta_3 BL + \beta_4 BH + \beta_5 TIJ + \beta_6 AGE \\ & + \beta_7 GENDER + \beta_8 INITW + u_{1i} + TIJ \times u_{2i} + e_{ij} \end{aligned}$$

where AL, AH, BL and BH are dummy variables, coded as follows

	AL	AH	BL	BH
Drug A, low dosage	1	0	0	0
Drug A, high dosage	0	1	0	0
Drug B, low dosage	0	0	1	0
Drug B, high dosage	0	0	0	1

In the above model $\beta_1, \beta_2, \beta_3,$ and β_4 represent the average group loss (or gain) in weight over the study period if we control for a subject's age (AGE), gender (GENDER), weight at the onset of the trial (INITW), and time (TIJ) at which the weight loss (y_{ij}) measurement was made.

Visual inspection of the estimated β -coefficients may point to significant differences between the different treatments. The construction of contrasts or linear functions of the parameters is a useful statistical analysis tool and enables the researcher to perform hypothesis testing concerning the equality of subsets of parameters.

In the example above, the fixed part of the model has 8 parameters $\beta_1, \beta_2, \ldots, \beta_8$. We may want to test the following 3 hypotheses:

$$\begin{aligned} H_{01} &: \beta_1 = \beta_2 \\ H_{02} &: \beta_1 = \beta_3 \\ H_{03} &: \beta_1 = \beta_4. \end{aligned}$$

Each of these hypotheses can alternatively be written as

$$\begin{aligned} H_{01} &: 1\beta_1 - 1\beta_2 + 0\beta_3 + 0\beta_4 + 0\beta_5 + 0\beta_6 + 0\beta_7 + 0\beta_8 = 0 \\ H_{02} &: 1\beta_1 + 0\beta_2 - 1\beta_3 + 0\beta_4 + 0\beta_5 + 0\beta_6 + 0\beta_7 + 0\beta_8 = 0 \\ H_{03} &: 1\beta_1 + 0\beta_2 + 0\beta_3 - 1\beta_4 + 0\beta_5 + 0\beta_6 + 0\beta_7 + 0\beta_8 = 0 \end{aligned}$$

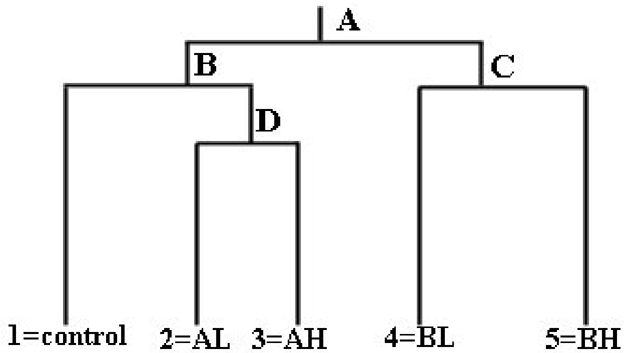
or, in matrix notation,

$$H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0},$$

where

$$\mathbf{C} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Suppose that an additional 100 subjects (the control group) are also assigned to the experiment, but each subject from this group receives a placebo. Suppose further that the 5 treatments are hypothesized to be related as described by the tree diagram



Here we can form the orthogonal contrasts:

	Treatments								
Contrast	**1**	**2**	**3**	**4**	**5**	**TIJ**	**AGE**	**GENDER**	**INITW**
A	1/3	1/3	1/3	-1/2	-1/2	0	0	0	0
B	1	-1/2	-1/2	0	0	0	0	0	0
C	0	0	0	1	-1	0	0	0	0
D	0	1	-1	0	0	0	0	0	0

A complex hypothesis about several elements of the vector of fixed coefficients $\boldsymbol{\beta}$ can be tested if use is made of a $p \times m$ contrast matrix **C**, with *p* the number of contrasts and *m* the number of fixed coefficients. The hypothesis is written in the form

$$\mathbf{C}\boldsymbol{\beta} = \mathbf{k}$$

,

where **k** is a known vector, usually **k** = **0**.

For large samples (see *e.g*. du Toit, 1993), $\mathbf{C}\hat{\boldsymbol{\beta}}$ has an approximate $N\left(\mathbf{C}\boldsymbol{\beta}, \mathbf{C}\boldsymbol{\Gamma}^{-1}\mathbf{C}^{'}\right)$ distribution, where $\boldsymbol{\Gamma} = Cov(\hat{\boldsymbol{\beta}})$. If the hypothesis $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{k}$ is true, it follows (see, *e.g*. Anderson (2003)), that

$$U = \left(\mathbf{C}\boldsymbol{\beta} - \mathbf{k}\right)^{'}\left[\mathbf{C}\boldsymbol{\Gamma}^{-1}\mathbf{C}^{'}\right]^{-1}\left(\mathbf{C}\boldsymbol{\beta} - \mathbf{k}\right)$$

follows an approximate χ^2-distribution with *p* degrees of freedom.

A set of $100\,(1-\alpha)\%$ simultaneous confidence intervals for the p elements of $\mathbf{C\beta}$ is given by the p intervals

$$\mathbf{c}_i^{'}\hat{\boldsymbol{\beta}} \pm \left[\mathbf{c}_i^{'}\boldsymbol{\Gamma}^{-1}\mathbf{c}_i \chi^2_{m,\alpha} \right]^{0.5},$$

where $p \le m$, $\mathbf{c}_i^{'}$ denotes the *i*-th row of $\mathbf{C}$ and $\chi^2_{m,\alpha}$ is the critical value of the χ^2 distribution with m degrees of freedom.

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