Two-way ANOVA allows to compare population means when the populations are classified according to two (categorical) factors.

Example. We might like to look at SAT scores of students who are male or female (first factor) and either have or have not had a preparatory course (second factor).

Example. A researcher wants to investigate the effects of the amounts of calcium and magnesium in a rat's diet on the rat's blood pressure. Diets including high, medium and low amounts of each mineral (but otherwise identical) will be fed to the rats. And after a specified time on the diet, the bood pressure wil be measured. Notice that the design includes nine different treatments because there are three levels to each of the two factors.

Advantages of the 2-way ANOVA

- usually have a smaller total sample size, since you're studying two things at once [rat diet example]
- removes some of the random variability (some of the random variability is now explained by the second factor, so you can more easily find significant differences)
- we can look at **interactions** between factors (a significant interaction means the effect of one variable changes depending on the level of the other factor).

Examples of (potential) interaction.

- Radon (high/medium/low) and smoking.

  High radon levels increase the rate of lung cancer somewhat. Smoking increases the risk of lung cancer. But if you are exposed to radon and smoke, then your lung cancer rates skyrocket. Therefore, the effect of radon on lung cancer rates is small for non-smokers but big for smokers. We can't talk about the effect of radon without talking about whether or not the person is a smoker.
- age of person (0-10, 11-20, 21+) and effect of pesticides (low/high)
- gender and effect of different legal drugs (different standard doses)

The Two-way ANOVA model

Suppose we have two factors with a levels for the first and b levels for the second. If we measure r individuals for each combination of factors (for a total of $n = abr$ data values) we have a design known as a *balanced* $a \times b$ *design*.

The mathematical model for this type of two-way ANOVA is

$$x_{ijk} = \underbrace{\mu}_{\text{grand mean}} + \underbrace{\alpha_i}_{\text{factor effect}} + \underbrace{\beta j}_{\text{factor effect}} + \underbrace{\gamma ij}_{\text{interaction effect}} + \underbrace{\epsilon_{ijk}}_{\text{residual}},$$

where $\epsilon_{ijk} \sim N(0, \sigma)$.

Think of this model in terms of making a prediction. Without knowledge of the two factors, our best guess is μ. If we know about the level of factor A, we can adjust the prediction by adding α_i. Similarly, knowlede of the level of factor B or knowledge about both provide additional information with which to modify our prediction. We still won't be correct every time, however, because there is random variation within each cell.

There are now three sets of hypotheses to test:

- H_0: no main effect for factor A (i.e. all ________ are 0)
- H_0: no main effect for factor B (i.e. all ________ are 0)
- H_0: no interaction effect (i.e. all ________ are 0)

Each of these hypothesis is judged on the basis of a separate F statistic.

As usual, we will use two-way ANOVA provided it is reasonable to assume normal group distributions and the ratio of the largest group standard deviation to the smallest group standard deviation is at most 2.

Two-way ANOVA table

Below is the outline of an $a \times b$ ANOVA table with two factors: A (a levels) and B (b levels). Recall that $n = abr$, where r is the number of replicates (number of individuals with each combination of levels of the two factors).

Source	df	SS	MS	F	p-value
A	$a-1$	SSA	MSA	MSA/MSE	
B	$b-1$	SSB	MSB	MSB/MSE	
A $\times$ B	$(a-1)(b-1)$	SSAB	MSAB	MSAB/MSE	
Error	$n-ab$	SSE	MSE		
Total	$n-1$	SST			

The general layout of the ANOVA table should be familiar to us from the ANOVA tables we have seen for regression and one-way ANOVA. Notice that this time we are dividing the variation into four components:

1.

2.

3.

4.

Interaction plots: looking for main effects and interactions

Example. Looking at the mean BC (Being Cautious score) after classifying people by diagnosis (anxiety, depression, CDFS/Court referred) and prior abuse (yes/no) .

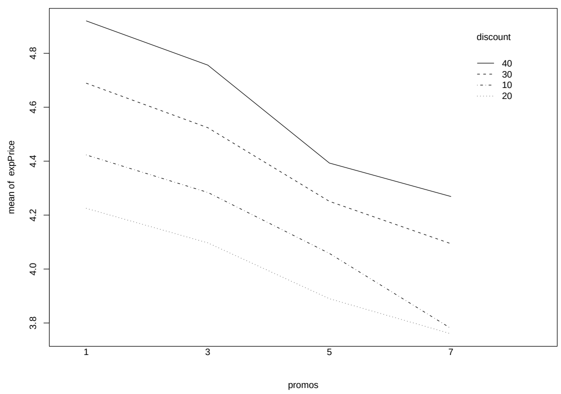
Diagnosis	Abused	Not abused	Row Mean
Anxiety	24.7	18.2	21.2
Depression	27.7	23.7	26.6
DCFS/Court Referred	29.8	16.4	20.8
Col Mean	27.1	19.4	

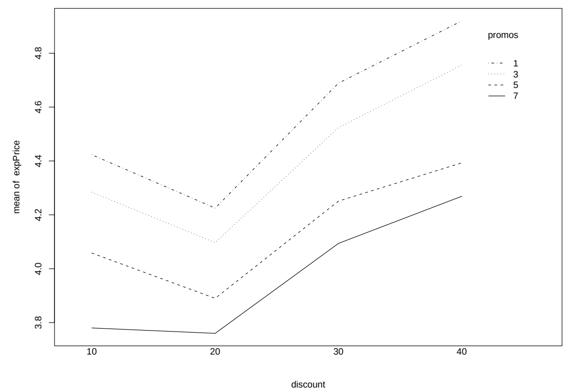
Here is the ANOVA table:

```
             df   SS        MS        F     p-value
Diagnosis    2    222.3     111.15   2.33    .11
Ever abused  1    819.06    819.06  17.2     .0001*
D * E        2    165.2      82.60   1.73    .186
Error        62  2958.0      47.71
Total        67
```

We can make two different interaction plots from the summary table above:

Example. Promotional fliers. [Page 821 in Moore/McCabe]






```
Anova(lm(expPrice ~ promos*discount))
Anova Table (Type II tests)

Response: expPrice
                Sum Sq  Df F value Pr(>F)
promos            8.36   3   47.73 <2e-16 ***
discount          8.31   3   47.42 <2e-16 ***
promos:discount   0.23   9    0.44   0.91
Residuals         8.41 144
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Means:
      discount
promos    10    20    30    40
     1 4.423 4.225 4.689 4.920
     3 4.284 4.097 4.524 4.756
     5 4.058 3.890 4.251 4.393
     7 3.780 3.760 4.094 4.269

Standard Deviations:

      discount
promos      10      20      30      40
     1 0.18476 0.38561 0.23307 0.15202
     3 0.20403 0.23462 0.27073 0.24291
     5 0.17599 0.16289 0.26485 0.26854
     7 0.21437 0.26179 0.24075 0.26992
```