Chapter 8
Factorial ANOVA: Higher order ANOVAs

Factorial ANOVA
Higher order ANOVAs

1. Three-way ANOVA
   - A three-way analysis of variance has three independent variables
     - Factor A with *a* levels
     - Factor B with *b* levels
     - Factor C with *c* levels

   - All of the procedures we developed for a two-way ANOVA can be extended to a three-way ANOVA. The interpretation gets more difficult and the math is messier

   - For simplicity, we will examine the simplest three way ANOVA: 2*2*2 design
     - Factor A with 2 levels
     - Factor B with 2 levels
     - Factor C with 2 levels

     I will present the formulas in their general form, and will give an example of a more complex design at the conclusion

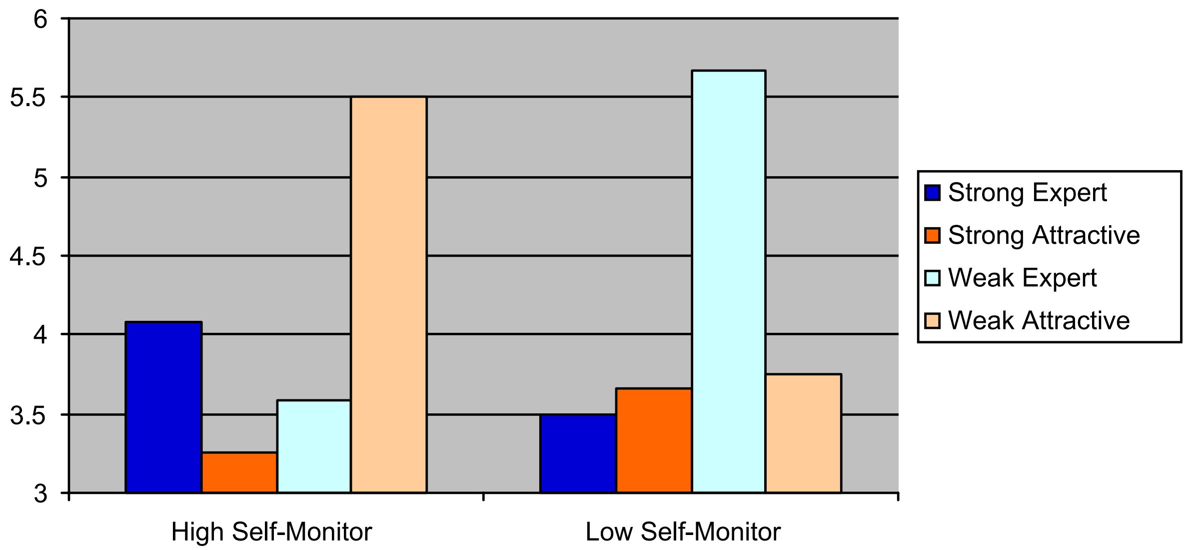
- An example of source expertise, source attractiveness, and the processing of persuasive information

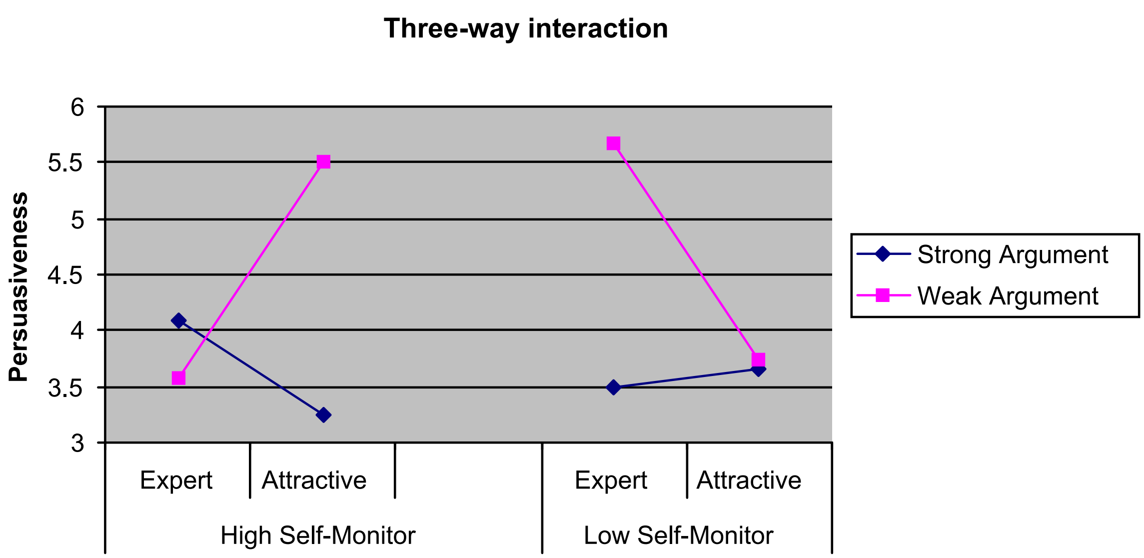
High Self-Monitors							
Strong Argument				*Weak Argument*			
Expert Source		*Attractive Source*		*Expert Source*		*Attractive Source*	
4	4	4	2	3	4	5	3
3	6	4	3	5	3	5	5
4	3	2	4	3	5	7	6
5	4	3	3	2	3	5	7
2	5	5	2	6	2	6	7
5	4	3	4	4	3	4	6

Low Self-Monitors							
Strong Argument				*Weak Argument*			
Expert Source		*Attractive Source*		*Expert Source*		*Attractive Source*	
3	1	5	2	5	5	6	4
5	5	4	4	6	6	4	3
5	3	3	4	4	4	4	4
4	4	2	3	7	6	2	2
3	3	4	4	6	7	4	3
2	4	6	3	7	5	5	4

$n_{jkl} = 12$

- Graphing three-factor ANOVA designs





• ANOVA Table for three-way ANOVA

		Sum of Squares	df	Mean Square	F	Sig.
Main Effects	(Combined)					
	MONITOR					
	STRENGTH					
	SOURCE					
2-Way Interactions	(Combined)					
	MONITOR * STRENGTH					
	MONITOR * SOURCE					
	STRENGTH * SOURCE					
3-Way Interactions	MONITOR * STRENGTH * SOURCE					
Model						
Residual						
Total						

2. Interpreting Effects

- Interpreting main effects

  - <u>The main effect of self-monitor</u> compares the levels of self-monitoring (high vs. low) after averaging over the levels of argument strength and source of argument

Self-Monitor	*Mean*	*Std Dev*	*N*
High	4.10	1.40	48
Low	4.15	1.43	48
SM Effect	*-0.05*		

  - <u>The main effect of argument strength</u> compares the levels of argument strength (strong vs. weak) after averaging over the levels of self-monitoring and source of argument

Strength	*Mean*	*Std Dev*	*N*
Strong	3.63	1.12	48
Weak	4.63	1.50	48
Strength Effect	*1.00*		

  - <u>The main effect of source of argument</u> compares the levels of source of argument (expert vs. attractive) after averaging over the levels of self-monitoring and argument strength

Source	*Mean*	*Std Dev*	*N*
Expert	4.21	1.43	48
Attractive	4.04	1.40	48
Source Effect	*0.17*		

- Interpreting two-way interactions
  - The self-monitor by strength of argument interaction examines the interaction of self-monitoring (high vs. low) and strength of argument (strong vs. weak) after averaging over the levels of source of argument
    Is the effect of self-monitoring the same at each level of strength of argument?
    Is the effect of strength of argument the same at each level of self-monitoring?

		Self-Monitoring	
		High	Low
Strength of Argument	Strong	3.67	3.58
	Weak	4.54	4.71
Strength Effect		*-0.87*	*-1.13*

$n_{jk} = 24$

  - The self-monitor by source of argument interaction examines the interaction of self-monitoring (high vs. low) and source of argument (expert vs. attractive) after averaging over the levels of strength of argument
    Is the effect of self-monitoring the same at each level of source of argument?
    Is the effect of source of argument the same at each level of self-monitoring?

		Self-Monitoring	
		High	Low
Source of Argument	Expert	3.83	4.58
	Attractive	4.38	3.71
Source Effect		*-0.55*	*0.87*

$n_{jl} = 24$

  - The strength of argument by source of argument interaction examines the interaction of strength of argument (strong vs. weak) and source of argument (expert vs. attractive) after averaging over the levels of self-monitoring
    Is the effect of strength of argument the same at each level of source of argument?
    Is the effect of source of argument the same at each level of strength of argument?

		Strength of Argument	
		Strong	Weak
Source of Argument	Expert	3.79	4.63
	Attractive	3.46	4.63
Strength Effect		*0.33*	*0.00*

$n_{kl} = 24$

- Interpreting three-way interactions
  - So far, the logic and interpretation of main effects and interactions is basically the same as the two-way design
  - Now, let's extend this logic to a three-way interaction

  - The self-monitor by strength of argument by source of argument interaction examines the interaction of self-monitoring (high vs. low) and strength of argument (strong vs. weak) and source of argument (expert vs. attractive)

    The three-way interaction addresses the following questions:
    - Is the strength of argument by source of argument interaction the same at each level of self-monitoring?

    - Is the self-monitor by strength of argument interaction the same at each level of source of argument?

    - Is the self-monitor by source of argument interaction the same at each level of strength of argument?

Let's examine each approach to the three-way interaction:
- Is the strength of argument by source of argument interaction the same at each level of self-monitoring?

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	4.08	3.58	3.50	5.67
	Attractive	3.25	5.50	3.67	3.75
Source Effect		0.83	-1.92	-0.17	1.92
Difference in Source Effect		2.75		-2.09	

- Is the self-monitor by strength of argument interaction the same at each level of source of argument?

$n_{jkl} = 12$		Expert Source		Attractive Source	
		Self-monitoring		Self-monitoring	
		High	Low	High	Low
Strength	Strong	4.08	3.50	3.25	3.67
	Weak	3.58	5.67	5.50	3.75
Strength Effect		0.50	-2.17	-2.25	-0.08
Difference in Strength Effect		2.67		-2.17	

- Is the self-monitor by source of argument interaction the same at each level of strength of argument?

$n_{jkl} = 12$		Strong Argument		Weak Argument	
		Source of Argument		Source of Argument	
		Expert	Attractive	Expert	Attractive
Self-Monitor	High	4.08	3.25	3.58	5.50
	Low	3.50	3.67	5.67	3.75
Monitoring Effect		0.58	-0.42	-2.09	1.75
Difference in Monitoring Effect		1		-3.84	

  - Each of the different ways of examining the three-way interaction will lead to the exact same analysis and conclusion. The combination you choose to present should be based on your theory/hypotheses

* Table summarizing the meaning of effects in an A*B*C Design
  (Maxwell & Delaney, 1990, p 318)

	Meaning
Main Effects	
A	Comparison of marginal means of Factor A, averaging over levels of B and C
B	Comparison of marginal means of Factor B, averaging over levels of A and C
C	Comparison of marginal means of Factor C, averaging over levels of A and B
Two-way Interactions	
A*B	Examines whether the A effect is the same at every level of B, averaging over levels of C Equivalently, examines whether the B effect is the same at every level of A, averaging over levels of C
A*C	Examines whether the A effect is the same at every level of C, averaging over levels of B Equivalently, examines whether the C effect is the same at every level of A, averaging over levels of B
B*C	Examines whether the B effect is the same at every level of C, averaging over levels of A Equivalently, examines whether the C effect is the same at every level of B, averaging over levels of A
Three-way Interaction	
A*B*C	Examines whether the two-way A*B interaction is the same at every level of C Equivalently, examines whether the two-way A*C interaction is the same at every level of B Equivalently, examines whether the two-way B*C interaction is the same at every level of A

3. Structural model & SS partitioning

- Structural Model for a three-way ANOVA

$$Y_{ijk} = MODEL + ERROR$$
$$Y_{ijkl} = \mu + \alpha_j + \beta_k + \gamma_l + (\alpha\beta)_{jk} + (\alpha\gamma)_{jl} + (\beta\gamma)_{kl} + (\alpha\beta\gamma)_{jkl} + \varepsilon_{ijkl}$$

Mean Model Components:

μ The overall mean of the scores

Main Effect Model Components:

α_j The effect of being in level *j* of Factor A

β_k The effect of being in level *k* of Factor B

γ_l The effect of being in level *l* of Factor C

Two-way Interaction Model Components:

$(\alpha\beta)_{jk}$ The effect of being in level *j* of Factor A and level *k* of Factor B

$(\alpha\gamma)_{jl}$ The effect of being in level *j* of Factor A and level *l* of Factor C

$(\beta\gamma)_{kl}$ The effect of being in level *k* of Factor B and level *l* of Factor C

Three-way Interaction Model Components:

$(\alpha\beta\gamma)_{jkl}$ The effect of being in level *j* of Factor A, level *k* of Factor B, and level *l* of Factor C

Error Components:

ε_{ijk} The unexplained part of the score

α_j : *The effect of being in level j of Factor A*

$$\alpha_j = \mu_{.j..} - \mu_{....}$$
$$\sum_{j=1}^{a} \alpha_j = 0$$

β_k : *The effect of being in level k of Factor B*

$$\beta_k = \mu_{..k.} - \mu_{....}$$
$$\sum_{k=1}^{b} \beta_k = 0$$

γ_l : *The effect of being in level l of Factor C*

$$\gamma_l = \mu_{...l} - \mu_{....}$$
$$\sum_{l=1}^{c} \gamma_l = 0$$

$(\alpha\beta)_{jk}$ *The effect of being in level j of Factor A and level k of Factor B*

$$(\alpha\beta)_{jk} = \mu_{.jk.} - (\mu_{....} + \alpha_j + \beta_k)$$
$$\sum_{j=1}^{a} (\alpha\beta)_{jk} = 0 \quad \textit{for each level of j}$$
$$\sum_{k=1}^{b} (\alpha\beta)_{jk} = 0 \quad \textit{for each level of k}$$

$(\alpha\gamma)_{jl}$ *The effect of being in level j of Factor A and level l of Factor C*

$$(\alpha\gamma)_{jl} = \mu_{.j\cdot l} - (\mu_{....} + \alpha_j + \gamma_l)$$
$$\sum_{j=1}^{a} (\alpha\gamma)_{jl} = 0 \quad \textit{for each level of j}$$
$$\sum_{l=1}^{c} (\alpha\gamma)_{jl} = 0 \quad \textit{for each level of l}$$

$(\beta\gamma)_{kl}$ *The effect of being in level k of Factor B and level l of Factor C*

$$(\beta\gamma)_{kl} = \mu_{..kl} - (\mu_{....} + \beta_k + \gamma_l)$$
$$\sum_{k=1}^{b} (\beta\gamma)_{kl} = 0 \quad \textit{for each level of k}$$
$$\sum_{l=1}^{c} (\beta\gamma)_{kl} = 0 \quad \textit{for each level of l}$$

$(\alpha\beta\gamma)_{jkl}$ *The effect of being in level j of Factor A, level k of Factor B, and level l of Factor C*

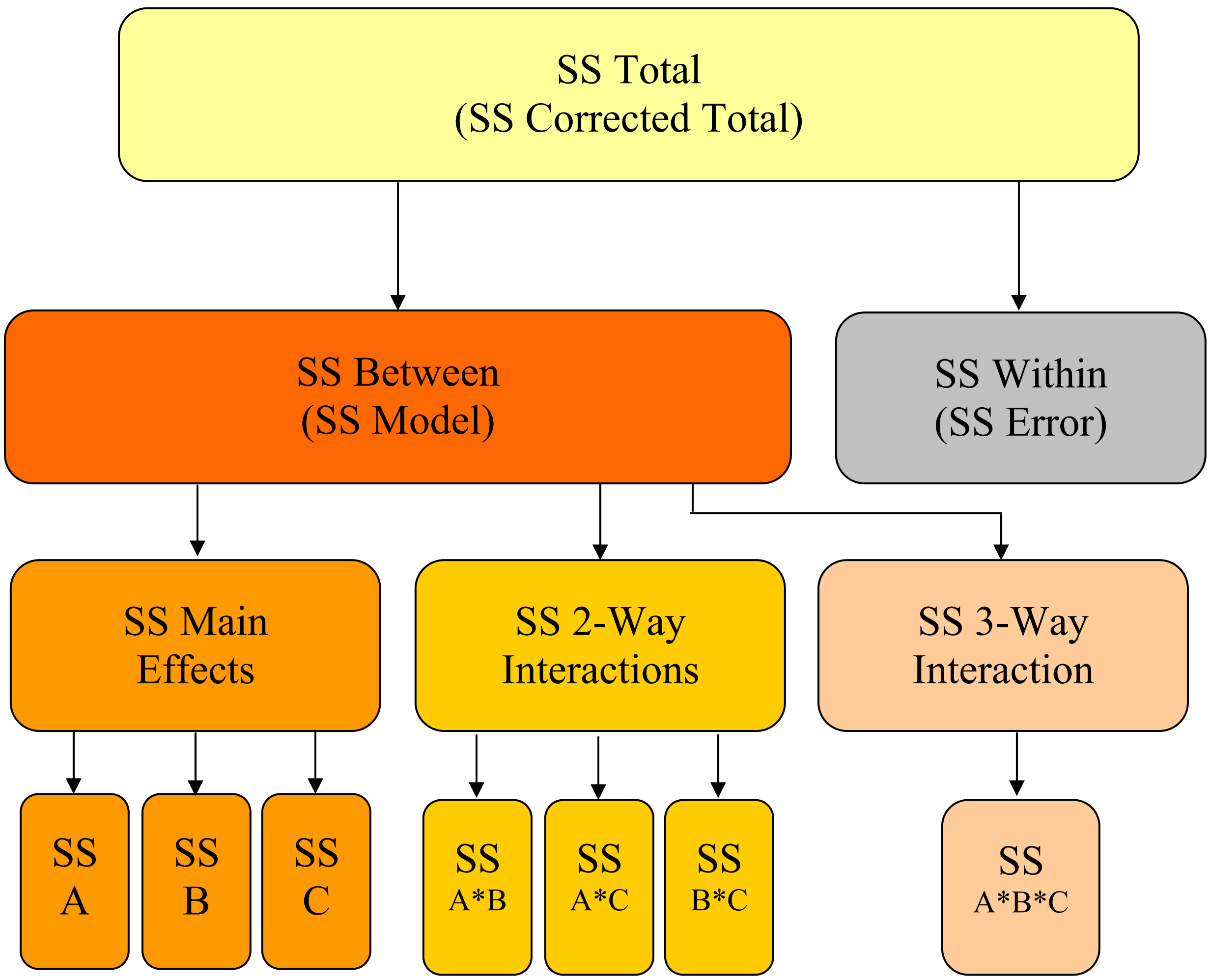
$$(\alpha\beta\gamma)_{jkl} = \mu_{jkl} - (\mu_{....} + \alpha_j + \beta_k + \gamma_l + \alpha\beta_{jk} + \alpha\gamma_{jl} + \beta\gamma_{kl})$$
$$\sum_{j=1}^{a} (\alpha\beta\gamma)_{jkl} = 0 \quad \textit{for each level of j} \qquad \sum_{k=1}^{b} (\alpha\beta\gamma)_{jkl} = 0 \quad \textit{for each level of k}$$
$$\sum_{l=1}^{c} (\alpha\beta\gamma)_{jkl} = 0 \quad \textit{for each level of l}$$

ε_{ijkl} *The unexplained part of the score*

$$\varepsilon_{ijkl} = Y_{ijkl} - MODEL$$
$$= Y_{ijkl} - \left(\mu + \alpha_j + \beta_k + \gamma_l + (\alpha\beta)_{jk} + (\alpha\gamma)_{jl} + (\beta\gamma)_{kl} + (\alpha\beta\gamma)_{jkl}\right)$$

- You should be able to compute and interpret each component of a three-way ANOVA model. In addition, you should be able to decompose each score into its structural model components

- Variance partitioning for a three-way ANOVA



- This SS partition only holds for balanced designs

- We showed the derivation of these SS formulas and how to compute them for the one-way and the two-way ANOVA case. The three-way formulas are extensions of these simpler formulas. You may find the formulas in any advanced ANOVA book (For example, see Kirk, 1995, p 441)

- The math works out nicely (as we would expect) so that if we take the ratio of the MS for a component of the model over the MS error, we obtain a valid test of the model component

- ANOVA table for three-way ANOVA

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Main effects					
Factor A	SSA	(a-1)	SSA/df_a	MSA/MSW	
Factor B	SSB	(b-1)	SSB/df_b	MSB/MSW	
Factor C	SSC	(c-1)	SSC/df_c	MSC/MSW	
Two-way Interactions					
A * B interaction	SSAB	(a-1)(b-1)	SSAB/df_{ab}	MSAB/MSW	
A * C interaction	SSAC	(a-1)(c-1)	SSAC/df_{ac}	MSAC/MSW	
B * C interaction	SSAB	(b-1)(c-1)	SSBC/df_{bc}	MSBC/MSW	
Three-way Interactions					
A * B * C interaction	SSABC	(a-1)(b-1)(c-1)	SSABC/df_{abc}	MSABC/MSW	
Model	SSBet	abc-1	SSB/df_{bet}		
Within	SSW	N-abc	SSW/df_w		
Total	SST	N-1			

- Using SPSS

```
UNIANOVA dv  BY monitor strength source
  /PRINT = DESCRIPTIVE.
```

Tests of Between-Subjects Effects

Dependent Variable: DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	72.833[a]	7	10.405	7.916	.000
Intercept	1633.500	1	1633.500	1242.778	.000
MONITOR	4.167E-02	1	4.167E-02	.032	.859
STRENGTH	24.000	1	24.000	18.259	.000
SOURCE	.667	1	.667	.507	.478
MONITOR * STRENGTH	.375	1	.375	.285	.595
MONITOR * SOURCE	12.042	1	12.042	9.161	.003
STRENGTH * SOURCE	.667	1	.667	.507	.478
MONITOR * STRENGTH * SOURCE	35.042	1	35.042	26.660	.000
Error	115.667	88	1.314		
Total	1822.000	96			
Corrected Total	188.500	95			

a. R Squared = .386 (Adjusted R Squared = .338)

- Summary of the results:

Main Effects:

Self-monitoring: $F(1, 88) = 0.03$, $p = .86$
Strength of Argument: $F(1, 88) = 18.26$, $p < .01$
Source of Argument: $F(1, 88) = 0.51$, $p = .48$

Two-way interactions:

Monitoring*Strength: $F(1, 88) = 0.29$, $p = .60$
Monitoring*Source: $F(1, 88) = 3.32$, $p = .02$
Strength*Source: $F(1, 88) = 0.51$, $p = .48$

Three-way interactions:

Monitoring*Strength*Source: $F(1, 88) = 26.66$, $p < .01$

4. Contrasts

- We can perform contrasts using the same method we developed for two-way ANOVA

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	2	5	6
	Attractive	3	4	7	8

```
if (monitor=1 and strength=1 and source=1) group = 1.
if (monitor=1 and strength=2 and source=1) group = 2.
if (monitor=1 and strength=1 and source=2) group = 3.
if (monitor=1 and strength=2 and source=2) group = 4.
if (monitor=2 and strength=1 and source=1) group = 5.
if (monitor=2 and strength=2 and source=1) group = 6.
if (monitor=2 and strength=1 and source=2) group = 7.
if (monitor=2 and strength=2 and source=2) group = 8.
```

- To test the main effect of self-monitoring:

Self-Monitoring	
High	Low
1	-1

$n_j = 48$

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	1	-1	-1
	Attractive	1	1	-1	-1

```
ONEWAY dv by group
 /CONT = 1 1 1 1 -1 -1 -1 -1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Self-Monitoring	-.1667	.93609	-.178	88	.859

$t(88) = -.18, p = .86$

- To test the main effect of strength of argument:

Strength of Argument	
Strong	Weak
1	-1

$n_k = 48$

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	-1	1	-1
	Attractive	1	-1	1	-1

```
ONEWAY dv by group
/CONT = 1 -1 1 -1 1 -1 1 -1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Strength	-4.0000	.93609	-4.273	88	.000

$t(88) = -4.28, p < .01$

- To test the main effect of source of argument:

Source of Argument	
Expert	Attractive
1	-1

$n_l = 48$

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	1	1	1
	Attractive	-1	-1	-1	-1

```
ONEWAY dv by group
 /CONT = 1 1 -1 -1 1 1 -1 -1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Source	.6667	.93609	.712	88	.478

$t(88) = 0.72, p = .48$

- To test the monitoring by strength interaction:

$n_{jk} = 24$		Self-Monitoring	
		High	Low
Strength of Argument	Strong	1	-1
	Weak	-1	1

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	-1	-1	1
	Attractive	1	-1	-1	1

```
ONEWAY dv by group
 /CONT = 1 -1 1 -1 -1 1 -1 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Monitoring * Strength	.5000	.93609	.534	88	.595

$t(88) = 0.53, p = .60$

- To test the monitoring by source interaction:

$n_{jl} = 24$		Self-Monitoring	
		High	Low
Source of Argument	Expert	1	-1
	Attractive	-1	1

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	1	-1	-1
	Attractive	-1	-1	1	1

```
ONEWAY dv by group
 /CONT = 1 1 -1 -1 -1 -1 1 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Monitoring * Source	-2.8333	.93609	-3.027	88	.003

$t(88) = -3.03, p < .01$

- To test the strength by source interaction:

$n_{kl} = 24$		Strength of Argument	
		Strong	Weak
Source of Argument	Expert	1	-1
	Attractive	-1	1

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	-1	1	-1
	Attractive	-1	1	-1	1

```
ONEWAY dv by group
 /CONT = 1 -1 -1 1 1 -1 -1 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Strength *Source	.6667	.93609	.712	88	.478

$t(88) = 0.72, p = .48$

- To test the monitoring by strength by source interaction:

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	-1	-1	1
	Attractive	-1	1	1	-1

```
ONEWAY dv by group
 /CONT = 1 -1 -1 1 -1 1 1 -1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	3-way	4.8333	.93609	5.163	88	.000

$t(88) = 5.16, p < .01$

- We can compute all the main effect and interaction tests with contrasts because for a 2*2*2 design, all the tests are single degree of freedom tests. For more complex a*b*c designs, omnibus tests with more than 1 degree of freedom can be performed using simultaneous tests of orthogonal contrasts.

- To compute these contrasts by hand, the formulas are simple generalizations of the two-way case:

$$\hat{\psi} = \sum_{l=1}^{c}\sum_{k=1}^{b}\sum_{j=1}^{a} c_{jkl}\overline{X}_{\cdot jkl} = c_{111}\overline{X}_{\cdot 111} + \ldots + c_{abc}\overline{X}_{\cdot abc}$$

$$\text{Std error }(\hat{\psi}) = \sqrt{MSW\sum_{l=1}^{c}\sum_{k=1}^{b}\sum_{j=1}^{a}\frac{c_{jkl}^2}{n_{jkl}}}$$

Where c_{jkl}^2 is the squared weight for each cell

n_{jkl} is the sample size for each cell

MSW is MSW from the omnibus ANOVA

$$t \sim \frac{\hat{\psi}}{\text{standard error}(\hat{\psi})} \qquad t_{observed} = \frac{\sum\sum\sum c_{jkl}\overline{X}_{\cdot jkl}}{\sqrt{MSW\sum\sum\sum\frac{c_{jkl}^2}{n_{jkl}}}}$$

$$\text{SS}\,\hat{\psi} = \frac{\hat{\psi}^2}{\sum\sum\sum\frac{c_{jkl}^2}{n_{jkl}}} \qquad F(1, dfw) = \frac{SSC/dfc}{SSW/dfw} = \frac{SSC}{MSW}$$

5. Planned & Post hoc tests

- Everything from the two-way design generalizes to the three-way design

- Planned tests can be corrected (if necessary) using Bonferroni

- Method for conducting post-hoc adjustments is same as for two-way design
    - Obtain observed t- or F-statistic by hand (or using SPSS, but discard printed p-value)
    - Look up critical value and compare to observed value

        - For Tukey's HSD using marginal means: $q(1-\alpha, d, \nu)$

          Where α = Familywise error rate
          d = Number of groups in the comparison
          ν = DFw = N-abc

        - For Tukey's HSD using all cell means: $q(1-\alpha, abc, \nu)$

          Where α = Familywise error rate
          abc = Number of cells in the design
          ν = DFw = N-abc

Compare $t_{observed}$ to $\frac{q_{crit}}{\sqrt{2}}$ or $F_{observed}$ to $\frac{(q_{crit})^2}{2}$

- For Scheffé using marginal means:

$$F_{Crit} = (d-1)F_{\alpha=.05;d-1,N-abc}$$

- For Scheffé using all cell means:

$$F_{Crit} = (a-1)(b-1)(c-1)F_{\alpha=.05;(a-1)(b-1)(c-1),N-abc}$$

Compare $F_{observed}$ to F_{crit}

6. Analyzing Effects

- Maxwell and Delaney's (1990) guidelines for analyzing effects in a three-factor design are considerably more complicated than for the two-factor design (recall p 7-59)!
- The principle remains the same. You must start with the highest order significant effect. You decompose these effects into simpler effects until you have an understanding of where the significant differences lie.

- Simple (interaction) effect
  - If you have a significant three-way interaction, then you need to examine the separate two-way interactions
    - The A*B interaction at each level of C or
    - The A*C interaction at each level of B or
    - The B*C interaction at each level of A

  - **APPROACH #1**: In our example, we have a significant three-way interaction, so let's examine the source of argument by strength of argument interaction at each level of self-monitoring

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	-1		
	Attractive	-1	1		

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert			1	-1
	Attractive			-1	1

- To examine the source of argument by strength of argument interaction for *high self-monitors*:

```
ONEWAY dv by group
  /CONT = 1 -1 -1 1 0 0 0 0 .
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	1	2.7500	.66191	4.155	88	.000

- To examine the source of argument by strength of argument interaction for *low self-monitors*:

```
ONEWAY dv by group
  /CONT = 0 0 0 0 1 -1 -1 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	1	-2.0833	.66191	-3.147	88	.002

- When the variances are homogeneous, these analysis can also be obtained with the MANOVA command.
  Note: You cannot obtain simple interaction effects with GLM

```
MANOVA dv  BY monitor (1,2) strength(1,2) source (1,2)
  /DESIGN  strength by source WITHIN monitor(1),
           strength by source WITHIN monitor(2),
           monitor * strength, monitor * source, monitor, strength, source.
```

Full Factorial Design	**Simple Effects Design**
Monitor	monitor
Strength	strength
Source	source
monitor * source	monitor * source
monitor * strength	monitor * strength
*strength * source* *monitor * strength * source*	source * strength WITHIN monitor (1) source * strength WITHIN monitor (2)

```
* * * * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * * * * *

 Tests of Significance for DV using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN+RESIDUAL          115.67      88      1.31
 STRENGTH BY SOURCE W      22.69       1     22.69     17.26      .000
 ITHIN MONITOR(1)
 STRENGTH BY SOURCE W      13.02       1     13.02      9.91      .002
 ITHIN MONITOR(2)
 MONITOR * STRENGTH          .38       1       .38       .29      .595
 MONITOR * SOURCE          12.04       1     12.04      9.16      .003
 MONITOR                     .04       1       .04       .03      .859
 STRENGTH                  24.00       1     24.00     18.26      .000
 SOURCE                      .67       1       .67       .51      .478

 (Model)                   72.83       7     10.40      7.92      .000
 (Total)                  188.50      95      1.98
```

- Because each of these separate two-way analyses are significant, we need to conduct additional follow-up tests

  - For high self-monitors: We can examine the effect of source of argument within each level of strength of argument
    (The main effect of source within high self-monitors and strong argument AND within high self-monitors and weak argument)

  - Alternately, for high self-monitors: We can examine the effect of strength of argument within each level of source of argument
    (The main effect of strength within high self-monitors and expert source AND within high self-monitors and attractive source)

  - These analyses should be repeated for low self-monitors

- **APPROACH #2**: Alternatively, we can examine the strength of argument by self-monitoring interaction at each level of source of argument

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1	-1	-1	1
	Attractive				

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert				
	Attractive	1	-1	-1	1

- Using contrasts:

```
ONEWAY dv by group
 /CONT = 1 -1 0 0 -1 1 0 0
 /CONT = 0 0 1 -1 0 0 -1 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Expert Source	2.6667	.66191	4.029	88	.000
	Attractive Source	-2.1667	.66191	-3.273	88	.002

- Using MANOVA:

```
MANOVA dv  BY monitor (1,2) strength(1,2) source (1,2)
  /DESIGN  strength by monitor WITHIN source(1),
           strength by monitor WITHIN source(2),
           source * strength, monitor * source, monitor, strength, source.
```

Full Factorial Design	**Simple Effects Design**
monitor	monitor
strength	strength
source	source
strength * source	strength * source
monitor * source	monitor * source
*monitor * strength*	
*monitor * strength * source*	monitor * strength WITHIN source (1)
	monitor * strength WITHIN source (2)

```
* * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * * *

 Tests of Significance for DV using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN+RESIDUAL          115.67      88      1.31
 STRENGTH BY MONITOR       21.33       1     21.33     16.23      .000
 WITHIN SOURCE(1)
 STRENGTH BY MONITOR       14.08       1     14.08     10.71      .002
 WITHIN SOURCE(2)
 SOURCE * STRENGTH           .67       1       .67       .51      .478
 MONITOR * SOURCE          12.04       1     12.04      9.16      .003
 MONITOR                     .04       1       .04       .03      .859
 STRENGTH                  24.00       1     24.00     18.26      .000
 SOURCE                      .67       1       .67       .51      .478

 (Model)                   72.83       7     10.40      7.92      .000
 (Total)                  188.50      95      1.98
```

- **APPROACH #3**: Alternatively, we can examine the source of argument by self-monitoring interaction at each level of strength of argument

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert	1		-1	
	Attractive	-1		1	

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert		1		-1
	Attractive		-1		1

- Using contrasts:

```
ONEWAY dv by group
 /CONT = 1 0 -1 0 -1 0 1 0
 /CONT = 0 1 0 -1 0 -1 0 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Strong Argument	1.0000	.66191	1.511	88	.134
	Weak Argument	-3.8333	.66191	-5.791	88	.000

- Using MANOVA:

```
MANOVA dv  BY monitor (1,2) strength(1,2) source (1,2)
  /DESIGN  monitor by source WITHIN strength(1),
           monitor by source WITHIN strength(2),
           monitor * strength, strength * source, monitor, strength, source.
```

Full Factorial Design	**Simple Effects Design**
monitor	monitor
strength	strength
source	source
strength * source	strength * source
monitor * strength	monitor * strength
*monitor * source* }	
*monitor * strength * source* }	monitor * source WITHIN strength (1)
	monitor * source WITHIN strength (2)

```
* * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * * *

 Tests of Significance for DV using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN+RESIDUAL          115.67      88      1.31
 MONITOR BY SOURCE WI       3.00       1      3.00      2.28      .134
 THIN STRENGTH(1)
 MONITOR BY SOURCE WI      44.08       1     44.08     33.54      .000
 THIN STRENGTH(2)
 MONITOR * STRENGTH          .38       1       .38       .29      .595
 STRENGTH * SOURCE           .67       1       .67       .51      .478
 MONITOR                     .04       1       .04       .03      .859
 STRENGTH                  24.00       1     24.00     18.26      .000
 SOURCE                      .67       1       .67       .51      .478

 (Model)                   72.83       7     10.40      7.92      .000
 (Total)                  188.50      95      1.98
```

- We should not take all three approaches; only one is necessary. The choice you make should be the one that makes the most sense for your theory/hypotheses

- For approach 3, we found
  - No significant self-monitoring by source interaction for strong messages, $F(1,88) = 2.28, p = .13$.
  - A significant self-monitoring by source interaction for weak messages, $F(1,88) = 33.54, p < .01$. We need to conduct follow-up tests to interpret this simple interaction effect. These tests are called simple, simple, main effects.

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert		1		
	Attractive		-1		

$n_{jkl} = 12$		High Self-Monitor		Low Self-Monitor	
		Strength of Argument		Strength of Argument	
		Strong	Weak	Strong	Weak
Source	Expert				-1
	Attractive				1

- Using Contrasts:

```
ONEWAY dv by group
 /CONT = 0 1 0 -1 0 0 0 0
 /CONT = 0 0 0 0 0 -1 0 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	High Monitor, Weak Message	-1.9167	.46804	-4.095	88	.000
	Low Monitor, Weak Message	-1.9167	.46804	-4.095	88	.000

- Using MANOVA

```
MANOVA dv  BY monitor (1,2) strength(1,2) source (1,2)
  /DESIGN  source WITHIN monitor (1) WITHIN strength(1),
           source WITHIN monitor (2) WITHIN strength(1),
           source WITHIN monitor (1) WITHIN strength(2),
           source WITHIN monitor (2) WITHIN strength(2),
           monitor * strength, monitor, strength.
```

```
* * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * * *

 Tests of Significance for DV using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN+RESIDUAL          115.67      88      1.31
 SOURCE WITHIN MONITO       4.17       1      4.17      3.17      .078
 R(1) WITHIN STRENGTH
 (1)
 SOURCE WITHIN MONITO        .17       1       .17       .13      .723
 R(2) WITHIN STRENGTH
 (1)
 SOURCE WITHIN MONITO      22.04       1     22.04     16.77      .000
 R(1) WITHIN STRENGTH
 (2)
 SOURCE WITHIN MONITO      22.04       1     22.04     16.77      .000
 R(2) WITHIN STRENGTH
 (2)
 MONITOR * STRENGTH          .38       1       .38       .29      .595
 MONITOR                     .04       1       .04       .03      .859
 STRENGTH                  24.00       1     24.00     18.26      .000

 (Model)                   72.83       7     10.40      7.92      .000
 (Total)                  188.50      95      1.98
```

- Using GLM

```
UNIANOVA  dv  BY monitor strength source
  /EMMEANS = TABLES(monitor*strength*source) COMPARE(source)
  /PRINT = DESCRIPTIVE .
```

Univariate Tests

Dependent Variable: dv

monitor	strength		Sum of Squares	df	Mean Square	F	Sig.
High	Strong	Contrast	4.167	1	4.167	3.170	.078
		Error	115.667	88	1.314		
	Weak	Contrast	22.042	1	22.042	16.769	.000
		Error	115.667	88	1.314		
Low	Strong	Contrast	.167	1	.167	.127	.723
		Error	115.667	88	1.314		
	Weak	Contrast	22.042	1	22.042	16.769	.000
		Error	115.667	88	1.314		

Each F tests the simple effects of source within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

- If these tests are planned or post-hoc, they need to be adjusted accordingly

- An alternative to the simple effect approach is the contrast-based approach.
  - The traditional approach conducts 7 uncorrected omnibus tests, so we are allowed 7 uncorrected planned contrasts. If you have more than 7 planned contrasts, you must use the Bonferroni correction.
  - Post-hoc tests can be conducted using Tukey HSD or Scheffé to keep $\alpha_{EW} = .05$

7. Effect sizes

- Formulas for partial omega-squared and r (for contrasts only) are easily adapted to a three-factor design:

$$\hat{\omega}^2_{(EFFECT)} = \frac{SS(effect) - [df(effect)]MSWithin}{SS(effect) + [N - df(effect)]MSWithin}$$

$$r = \sqrt{\frac{F_{contrast}}{F_{contrast} + df_{within}}} = \sqrt{\frac{t^2_{contrast}}{t^2_{contrast} + df_{within}}}$$

- For example, to compute the proportion of variance accounted for by the three-way interaction in our persuasion example

$$\hat{\omega}^2_{(A*B*C)} = \frac{SS(ABC) - [df(ABC)]MSWithin}{SS(ABC) + [N - df(ABC)]MSWithin}$$

$$\hat{\omega}^2_{(A*B*C)} = \frac{35.042 - (1)1.314}{35.042 + [96 - 1]1.314} = .21$$

$F(1,88) = 26.66, p < .001, \ \omega^2 = .21$

8. Higher-order ANOVA

- The logic we developed for two- and three-factor ANOVA can be easily extended to four-factor, five-factor and even higher order ANOVAs
- By now you have seen how the formulas generalize so that you can compute values for any order design

- Interpretation of a three-factor ANOVA is tricky enough. Things get very hairy for higher order ANOVAs.
  - o For example, a significant four-way interaction (A*B*C*D) indicates that the three way A*B*C interaction is not the same at each level of D or that the three way A*B*D interaction is not the same at each level of C or . . .
  - o We saw that to graph a three-way 2*2*2 interaction, we had to graph two separate two-way interactions
    - To graph a four-way 2*2*2*2 interaction, we would have to graph four separate two-way interactions!
    - To graph a five-way 2*2*2*2*2 interaction, we would have to graph eight separate two-way interactions!
  - o Remember when you design a study, you will need to be able to analyze, understand, and present the results. It is rare that a person can clearly present a four-way interaction in a manner that the audience can understand. Beware of conducting designs that are too complex!

- As the number of factors increases, the number of omnibus tests increases rapidly. Because the convention is to use $\alpha = .05$ for each omnibus test, the probability of making a type one error is high for a multi-factor ANOVA

			Number of			
Number of Factors	Main Effects	Two-way Interactions	Three-way Interactions	Four-way Interactions	Five-way Interactions	Total Number of tests
2	2	1				3
3	3	3	1			7
4	4	6	4	1		15
5	5	10	10	5	1	31

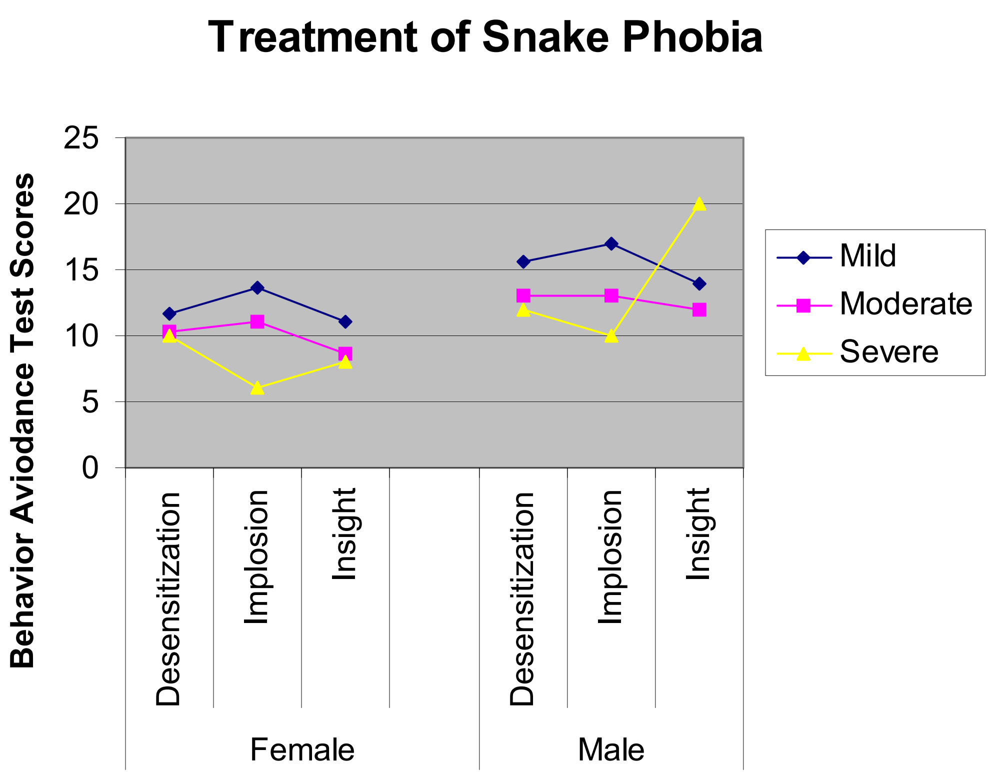
As a result, do not be surprised if you are asked to replicate the results of your multi-factor ANOVA.

9. Example: A 3*3*2 design

- Consider an experiment comparing three types of therapy for modifying snake phobia
    - Factor A – Degree of Phobia: Mild, Moderate, Severe
    - Factor B – Type of Therapy: Desensitization, Implosion, Insight
    - Factor C – Gender: Male, Female

    - DV = Post-test scores on the Behavioral Avoidance Test (higher scores indicate less phobia)

	Desensitization			**Implosion**			**Insight**		
	Mild	*Moderate*	*Severe*	*Mild*	*Moderate*	*Severe*	*Mild*	*Moderate*	*Severe*
Females	10	12	10	15	12	6	13	11	10
	12	9	11	12	10	7	9	7	6
	13	10	9	14	11	5	11	8	8
Males	16	11	12	17	14	10	16	10	11
	14	13	11	18	13	9	12	12	10
	17	15	13	16	12	11	14	14	9



- First, let's approach the analysis the traditional way:

```
UNIANOVA dv  BY treat phobia gender
  /PRINT = DESCRIPTIVE.
```

Tests of Between-Subjects Effects

Dependent Variable: DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	368.167[a]	17	21.657	9.356	.000
Intercept	7141.500	1	7141.500	3085.128	.000
TREAT	22.333	2	11.167	4.824	.014
PHOBIA	183.000	2	91.500	39.528	.000
GENDER	115.574	1	115.574	49.928	.000
TREAT * PHOBIA	39.333	4	9.833	4.248	.006
TREAT * GENDER	.259	2	.130	.056	.946
PHOBIA * GENDER	1.815	2	.907	.392	.679
TREAT * PHOBIA * GENDER	5.852	4	1.463	.632	.643
Error	83.333	36	2.315		
Total	7593.000	54			
Corrected Total	451.500	53			

a. R Squared = .815 (Adjusted R Squared = .728)

- We have a treatment by phobia interaction, $F(4, 36) = 4.25, p = .006$
- We have a main effect of gender, $F(1, 36) = 49.93, p < .001$
    - We also have main effects for treatment and for phobia, but we should refrain from interpreting them because of the higher order interaction
    - We may interpret the main effect of gender because gender is not involved in any higher order interactions

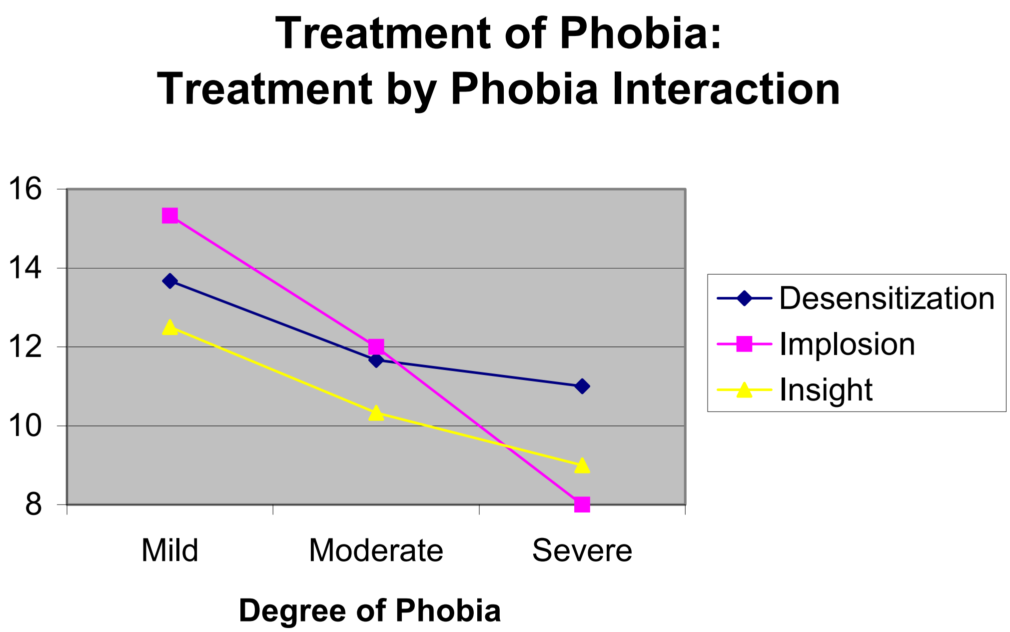
- Let's start with the main effect of gender. This analysis reveals the effect of gender averaging across type of treatment and severity of phobia.

Gender	*Mean*	*Std Dev*	*N*
Female	10.04	2.52	27
Male	12.96	2.56	27
Gender Effect	*-2.92*		

- This analysis tells us that men show less post-test phobia than women, *averaging across type of treatment and severity of phobia*.

  Because this test has only 1 df, no follow-up tests are necessary

- Now, let's turn to the treatment by phobia interaction. This analysis tells us that the main effect for treatment differs by the degree of phobia, *averaging across gender*.



		Degree of Phobia		
		Mild	Moderate	Severe
Treatment	Desens.	13.67	11.67	11.00
	Implosion	15.33	12.00	8.00
	Insight	12.50	10.33	9.00

  - To understand this interaction, we can examine the simple effect of degree of phobia within each type of treatment

		Degree of Phobia		
		Mild	Moderate	Severe
Treatment	Desens.	13.67	11.67	11.00
	Implosion	15.33	12.00	8.00
	Insight	12.50	10.33	9.00

According to Maxwell & Delaney's guidelines, we need to use the Bonferroni adjustment:

$$\alpha_{FW} = \frac{.05}{3} = 0.0167$$

- These simple effect tests will be two-degrees of freedom tests. We can not test these hypotheses with a single contrast. If we have homogeneous variances, we can use the MANOVA command.

```
MANOVA dv  BY gender (1,2) treat(1,3) phobia (1,3)
  /DESIGN  phobia WITHIN treat (1), phobia WITHIN treat (2),
           phobia WITHIN treat (3), treat * phobia *gender, gender * treat,
           gender * phobia, gender, treat .
```

```
* * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * * *
```

Tests of Significance for DV using UNIQUE sums of squares

Source of Variation	SS	DF	MS	F	Sig of F
WITHIN+RESIDUAL	83.33	36	2.31		
PHOBIA WITHIN TREAT(1)	**23.11**	**2**	**11.56**	**4.99**	**.012**
PHOBIA WITHIN TREAT(2)	**161.78**	**2**	**80.89**	**34.94**	**.000**
PHOBIA WITHIN TREAT(3)	**37.44**	**2**	**18.72**	**8.09**	**.001**
TREAT * PHOBIA * GENDER	5.85	4	1.46	.63	.643
GENDER * TREAT	.26	2	.13	.06	.946
GENDER * PHOBIA	1.81	2	.91	.39	.679
GENDER	115.57	1	115.57	49.93	.000
TREAT	22.33	2	11.17	4.82	.014
(Model)	368.17	17	21.66	9.36	.000
(Total)	451.50	53	8.52		

- Simple effect of degree of phobia for participants who received desensitization treatment:

$F(2,36) = 4.99, p = 0.012$

$F(2,36) = 4.99, p < 0.05$ (with Bonferroni correction)

- Simple effect of degree of phobia for participants who received implosion treatment:

$F(2,36) = 34.94, p < .001$

$F(2,36) = 34.94, p < 0.05$ (with Bonferroni correction)

- Simple effect of degree of phobia for participants who received insight treatment:

$F(2,36) = 8.09, p = .001$

$F(2,36) = 8.09, p < 0.05$ (with Bonferroni correction)

- We have found significant simple effects of degree of phobia for participants who received desensitization, implosion or the insight treatments. These are omnibus tests, so we need to do Tukey post-hoc tests (with $\alpha = .0167$) to identify the differences.

```
if (treat=1 and phobia=1 and gender=1) group = 1.
if (treat=1 and phobia=2 and gender=1) group = 2.
if (treat=1 and phobia=3 and gender=1) group = 3.
. . .
. . .
if (treat=3 and phobia=1 and gender=2) group = 16.
if (treat=3 and phobia=2 and gender=2) group = 17.
if (treat=3 and phobia=3 and gender=2) group = 18.
```

```
ONEWAY dv by group
 /CONT = -1 1 0 0 0 0 0 0 0 -1 1 0 0 0 0 0 0 0
 /CONT = -1 0 1 0 0 0 0 0 0 -1 0 1 0 0 0 0 0 0
 /CONT = 0 -1 1 0 0 0 0 0 0  0 -1 1 0 0 0 0 0 0
 /CONT = 0 0 0 -1 1 0 0 0 0  0 0 0 -1 1 0 0 0 0
 /CONT = 0 0 0 -1 0 1 0 0 0  0 0 0 -1 0 1 0 0 0
 /CONT = 0 0 0 0 -1 1 0 0 0  0 0 0 0 -1 1 0 0 0
 /CONT = 0 0 0 0 0 0 -1 1 0  0 0 0 0 0 0 -1 1 0
 /CONT = 0 0 0 0 0 0 -1 0 1  0 0 0 0 0 0 -1 0 1
 /CONT = 0 0 0 0 0 0 0 -1 1  0 0 0 0 0 0 0 -1 1
```

(Note: Ignore Significance levels)

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	1	-4.0000	1.75682	-2.277	36	.029
	2	-5.3333	1.75682	-3.036*	36	.004
	3	-1.3333	1.75682	-.759	36	.453
	4	-6.6667	1.75682	-3.795*	36	.001
	5	-14.6667	1.75682	-8.348*	36	.000
	6	-8.0000	1.75682	-4.554*	36	.000
	7	-4.3333	1.75682	-2.467	36	.019
	8	-7.0000	1.75682	-3.984*	36	.000
	9	-2.6667	1.75682	-1.518	36	.138

- For Tukey's HSD following simple effects: $q(1-\alpha, r, \nu)$

  Where α = Familywise error rate

  r = Number of groups in the comparisons

  ν = DFw = $N-abc$

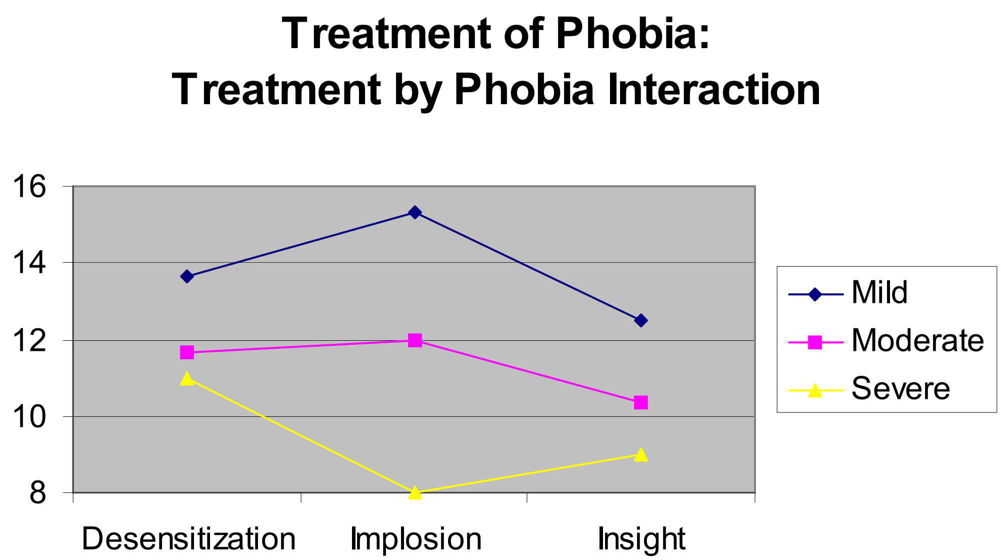
$q(.9833,3,36) = 4.11$

Compare $t_{observed}$ to $t_{critical} = \frac{4.11}{\sqrt{2}} = 2.91$

- We end up with the following description of an interaction:
    - There is a simple effect of degree of phobia for participants who received desensitization and for insight treatment. Tukey post-hoc tests revealed that treatment is significantly better for mild cases than severe cases.
    - There is a simple effect of degree of phobia for participants who received implosion treatment. Tukey post-hoc tests revealed mild phobic responded better than moderate phobic who responded better than severe phobics (with all pairwise differences significant)

- Remember, we also could have decompose the treatment by phobia interaction by examining the simple effect of treatment within each degree of phobia

    (But this analysis is left as an exercise for the reader)



- However, notice how much easier these results would have been to explain had the treatment by phobia interaction not been significant! (We would be left with three main effects!)
- The moral of the story is that you should not just add extra factors "just to see what might happen." You want to design as concise a study as possible while still testing your hypotheses.