# CHARACTERIZING GRAPHS OF ORDER N WHOSE SUM OF STRONG EFFICIENT DOMINATION NUMBER AND CHROMATIC NUMBER IS N + 1

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**ABSTRACT**

Let G = (V,E) be a graph. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every v ∈ V(G), $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$), where $N_s(v) = \{u \in V(G): uv \in E(G), \deg u \geq \deg v\}$ and $N_w(v) = \{u \in V(G) : uv \in E(G), \deg v \geq \deg u\}$, $N_s[v] = N_s(v) \cup \{v\}$, ($N_w[v] = N_w(v) \cup \{v\}$). The minimum cardinality of a strong (weak) efficient dominating set of G is called the strong (weak) efficient domination number of G and is denoted by $\gamma_{se}(G)$ ($\gamma_{we}(G)$). Let G be a graph on n vertices. In this paper, graphs for which the sum of strong efficient domination number and chromatic number equal to n+1 are characterized.

**Key words:** Strong efficient dominating sets, Strong efficient domination number and chromatic number

**AMS Subject classification:** (2010): 05C69

## 1. Introduction:

Throughout this paper, only finite, undirected and simple graphs are considered. Let G = (V,E) be a graph. A subset S of V(G) of a graph G is called a dominating set of G if every vertex in V(G) \ S is adjacent to a vertex in S [9]. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. Sampathkumar and Pushpalatha introduced the concepts of strong and weak domination in graphs [8]. A subset S of V(G) is called a strong dominating set of G if for every $v \in V - S$ there exists a $u \in S$ such that u and v are adjacent and deg u ≥ deg v. A subset S of V(G) is called an efficient dominating set of G if for every $v \in V(G)$, $|N[v] \cap S| = 1$[1,2]. The concept of strong (weak) efficient domination in graphs was introduced by Meena, Subramanian and Swaminathan [4]. A subset S of V(G) is called a strong (weak) efficient dominating set of G if for every $v \in V(G)$, $|N_s[v] \cap S| = 1$ ($|N_w[v] \cap S| = 1$). $N_s(v) = \{ u \in V(G) : uv \in E(G), \deg(u) \geq \deg(v) \}$.The minimum cardinality of a strong (weak) efficient dominating set is called strong (weak) efficient domination number and is denoted by $\gamma_{se}(G)$($\gamma_{we}(G)$). A graph G is strong efficient if there exists a strong efficient dominating set of G. An *n* - colouring of a graph *G* uses n colours**.** The chromatic number $\chi(G)$ is defined as the minimum *n* for which *G* has an n - colouring. Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and graph theoretic parameter and characterized the extremal graphs. In [6] Paulraj Joseph J and Arumugam S proved that $\gamma + \chi \leq n$. In [5], they proved that $\gamma + \chi \leq n+1$ and characterized the classes for which the upper bound is obtained. In [7], Paulraj. Joseph and Mahadevan G characterized the classes of graphs for which the sum of complementary connected domination number and chromatic number less than or equal to 2n – 6. Motivated by these results, an attempt has been made to find the sum of strong efficient domination number and chromatic number of a strong efficient graph. In this paper, the graphs for which the sum of strong efficient domination number and chromatic number equal to n+1 are characterized. For all graph theoretic terminologies and notations, Harary [3] is followed.

**2. Main Result:**

**Theorem 2.1:** Let G = (V, E) be a simple graph on vertices. Let $\chi(G) \geq n - 2$. $\gamma_{se}(G) + \chi(G) = n + 1$ if and only if one of the following holds.

i. $G = K_n$.

ii. G is a split graph G = (C, I), with $C(G) = K_{n-2}$ or $K_{n-1}$ ; I, $I(G) = \overline{K}_2$ or $K_1$ and if $C(G) = K_{n-2}$ and $I(G) = \overline{K}_2$ then every vertex of C(G)is adjacent to one vertex of I(G), each vertex of I(G) is adjacent with atleast one vertex of C(G) and N(I(G)) = C(G). If $C(G) = K_{n-1}$ and $I(G) = K_1$ then $N(I(G)) = \phi$.

iii. G is a split graph G = (C, I) where $C = K_{n-3}$ and |I| = 3. G contains an isolate which obviously belongs to I and other two vertices of I have non-empty disjoint neighbourhoods in C whose union is C.

iv. G is a split graph G = (C, I) where $C = K_{n-2}$ and both the vertices of I are isolates of G (That is $G = K_{n-2} \cup 2K_1$).

**Proof: Case (i):** Let $\chi(G) = n$. Then $G = K_n$. Therefore $\gamma_{se}(G) = 1$. Hence $\gamma_{se}(G) + \chi(G) = n + 1$.

**Case (ii):** Let $\chi(G) = n - 1$. Then G has a ψ partition $\prod = \{V_1, V_2, \dots, V_{n-1}\}$. Each $V_i$ has atleast one element and hence n – 2 elements of ∏ have one element each and the remaining has two elements. Therefore G contains a clique $C = K_{n-2}$ with the remaining two elements u, v being independent. Also u is adjacent to t vertices of C and v is adjacent to s vertices of C, $1 \leq t, s \leq n - 2$. If C has a vertex which is adjacent to both u and v then G has a full degree vertex. Hence $\gamma_{se}(G) = 1$. Thus $\gamma_{se}(G) + \chi(G) = n$, a contradiction. Therefore any vertex of C is adjacent with atmost one of u, v. Therefore G is obtained from $K_{n-2}$ by adding two vertices u, v which are not adjacent. $N(u) \cap N(v) \cap V(K_{n-2}) = \phi$. $[N(u) \cap V(K_{n-2})] \cup [N(v) \cap V(K_{n-2})] = V(K_{n-2})$.

Conversely if G is obtained in the above manner then $\gamma_{se}(G) + \chi(G) = n + 1$. If $N(u) \cap V(K_{n-2}) = \phi$ then u is an isolate and $G = K_{n-1} \cup K_1$. $\chi(G) = n - 1$, $\gamma_{se}(G) = 2$ and hence $\gamma_{se}(G) + \chi(G) = n + 1$.

Therefore $\gamma_{se}(G) + \chi(G) = n + 1$with $\chi(G) = n - 1$ if and only if G is a split graph, G = (C, I) with $C(G) = K_{n-2}$ or $K_{n-1}$; I, $I(G) = \overline{K}_2$ or $K_1$ and if $C(G) = K_{n-2}$ and $I(G) = \overline{K}_2$ then every vertex of C(G) is adjacent to atmost one vertex of I(G); each vertex of I(G) is adjacent with atleast one vertex of C(G) and N(I(G)) = C(G). If $C(G) = K_{n-1}$ and $I(G) = K_1$ then $N(I(G)) = \phi$.

**Case (iii):** Let $\chi(G) = n - 2$. Let $\prod = \{V_1, V_2, \dots, V_{n-2}\}$ be a chromatic partition of G. Since each $V_i$ is non-empty, there may be exactly one set containing three elements or two sets containing two elements each with the remaining sets being singletons.

**Subcase (i):** Let $|V_i| = 1$, $1 \leq i \leq n - 3$ and $|V_{n-2}| = 3$. Let $V_i = \{u_i\}$, $1 \leq i \leq n - 3$ and $V_{n-2} = \{u_{n-2}, u_{n-1}, u_n\}$. Since $\prod$ is a chromatic partition, $u_1, u_2, \dots, u_{n-3}$ form a complete subgraph $K_{n-3}$. Every vertex $u_i$, $1 \leq i \leq n - 3$ is adjacent with at least one vertex of $V_{n-2}$. Let $N(u_{n-2}) \cap V(K_{n-3}) = T_1$, $N(u_{n-1}) \cap V(K_{n-3}) = T_2$ and $N(u_n) \cap V(K_{n-3}) = T_3$. $T_1 \cup T_2 \cup T_3 = V(K_{n-3})$. If $T_1 \cap T_2 \cap T_3 \neq \phi$ then $\gamma_{se}(G) = 1$ and $\gamma_{se}(G) + \chi(G) = n - 1$, a contradiction.

**Subsubcase (i):** $T_1 \neq \phi$, $T_2 \neq \phi$ and $T_3 = \phi$. Then $u_3$ is an isolate. Let $T_1 \cap T_2 = \phi$. $\gamma_{se}(G) = 3$, since any $\gamma_{se}$ – set of G contains a vertex of $T_1$ and $u_{n-1}$, $u_n$ or contains a vertex of $T_2$ and $u_{n-2}$, $u_n$. Therefore $\gamma_{se}(G) + \chi(G) = 3 + n - 2 = n + 1$. G is a split graph (C, I) where C is $K_{n-3}$ and |I| = 3. G contains an isolate vertex which obviously belongs to I. The other two vertices of I have non-empty disjoint neighbourhoods in C whose union is C. If $T_1 \cap T_2 \neq \phi$, then $\gamma_{se}(G) = 2$ and hence $\gamma_{se}(G) + \chi(G) = n$, a contradiction.

**Subsubcase (ii):** $T_1, T_2, T_3 \neq \phi$. If $T_1 \cap T_2 \neq \phi$, then $\gamma_{se}(G) = 2$ and hence $\gamma_{se}(G) + \chi(G) = n$, a contradiction. Therefore $T_1, T_2, T_3$ are pair wise disjoint. $\gamma_{se}(G) = 3$, since any $\gamma_{se}$ – set of G contains one vertex each from $T_1$, $T_2$, and $T_3$ and deg $u_j \leq n - 5 <$ deg $u_i$ for every $j \in \{n-2, n-1, n\}$ and for every i, $1 \leq i \leq n - 3$. Therefore $\gamma_{se}(G) + \chi(G) = n + 1$. G is a split graph (C, I) where C is $K_{n-3}$ and $|I| = 3$ and three vertices of I have non-empty disjoint neighbourhoods in C whose union is C.

**Subsubcase (iii):** $T_1 = \phi = T_2$, $T_3 \neq \phi$. In this case $u_n$ is adjacent with every vertex $u_i$, $1 \leq i \leq n - 3$. Therefore $u_1, u_2, \dots, u_{n-3}, u_n$ form a complete subgraph $K_{n-2}$ with $u_{n-1}$, $u_{n-2}$ being isolate. Therefore $\gamma_{se}(G) = 3$. Any $\gamma_{se}$ – set of G contains a vertex from $K_{n-2}$ and the two vertices $u_{n-2}$ and $u_{n-1}$. Therefore $\gamma_{se}(G) + \chi(G) = n + 1$. Hence G is a split graph (C, I) where C is $K_{n-2}$ and I contains two vertices both of which are isolates of G.

**Subcase (ii):** Let $|V_i| = 1$, $1 \leq i \leq n - 4$ and $|V_{n-3}| = |V_{n-2}| = 2$. Let $V_i = \{u_i\}$, $1 \leq i \leq n - 4$. $V_{n-3} = \{u_{n-3}, u_{n-2}\}$ and $V_{n-2} = \{u_{n-1}, u_n\}$ Since $\prod = \{V_1, V_2, \dots, V_{n-2}\}$ is a $\chi$ – partition, $u_1, u_2, \dots, u_{n-4}$ form a complete subgraph $K_{n-4}$. Every $u_i$, $1 \leq i \leq n - 4$ is adjacent with atleast one vertex each of $V_{n-3}$, $V_{n-2}$. Atleast one vertex of $V_{n-3}$ is adjacent with a vertex of $V_{n-2}$. deg $u \geq n - 5 + 2 = n - 3$ for any vertex u in $K_{n-4}$. deg $u_{n-3}$, deg $u_{n-2}$, deg $u_{n-1}$, deg $u_n \leq n - 2$. deg $u_{n-i}$, $0 \leq i \leq 3$ is equal to $n - 2$ if and only if $u_{n-i}$ is adjacent with every vertex except the one which is independent with $u_{n-i}$. If for one vertex in $V_{n-3}$ and one vertex in $V_{n-2}$ are of degree $n - 2$, then every vertex in $K_{n-4}$ is of degree atleast $n - 3$. If $u_{n-3}$ is of degree $n - 2$, then $\{u_{n-3}, u_{n-2}\}$ is a $\gamma_{se}$ – set of G provided any neighbou.r of $u_{n-2}$ is of degree greater than deg $u_{n-2}$. The possible three element sets which may be $\gamma_{se}$- sets are $\{u, u_{n-3}, u_{n-2}\}$, $\{u, u_{n-3}, u_{n-1}\}$, $\{u, u_{n-3}, u_n\}$, $\{u, u_{n-2}, u_{n-1}\}$, $\{u, u_{n-2}, u_n\}$, $\{u, u_{n-1}, u_n\}$, $\{u_{n-3}, u_{n-2}, u_{n-1}\}$, $\{u_{n-3}, u_{n-2}, u_n\}$, $\{u_{n-2}, u_{n-1}, u_n\}$, $\{u_{n-3}, u_{n-1}, u_n\}$. Since $u_{n-3}$ is of degree $n - 2$, $u_{n-3}$ is adjacent with every vertex of G except $u_{n-2}$. Therefore those sets containing u and $u_{n-3}$ or $u_{n-3}$ and $u_{n-1}$ or $u_{n-3}$ and $u_n$ are not independent. The left out four sets are $\{u, u_{n-2}, u_{n-1}\}$, $\{u, u_{n-2}, u_n\}$, $\{u, u_{n-1}, u_n\}$ and $\{u_{n-2}, u_{n-1}, u_n\}$.

Consider $\{u, u_{n-2}, u_{n-1}\}$. $u_{n-3}$ is of degree n – 2 and is strongly dominated by either u or $u_{n-1}$. Therefore deg u or deg $u_{n-1} \geq n - 2$. Since $\{u, u_{n-2}, u_{n-1}\}$ is an independent set, deg $u \leq n - 3$ and deg $u_{n-1} \leq n - 3$. Therefore $u_{n-3}$ cannot be strongly dominated by $\{u, u_{n-2}, u_{n-1}\}$. Therefore $\{u, u_{n-2}, u_{n-1}\}$ is not a strong efficient dominating set of G. There exist no strong efficient dominating set of G of cardinality 3. Therefore $\gamma_{se}(G) + \chi(G) \neq n + 1$. Hence deg $u \leq n - 3$. By the same reasoning, deg $u_{n-2}$, deg $u_{n-1}$, deg $u_n \leq n - 3$.

Suppose deg u = n – 2. The possible three element sets which may be $\gamma_{se}$ – sets of G are $\{u, u_{n-3}, u_{n-2}\}$, $\{u, u_{n-3}, u_{n-1}\}$, $\{u, u_{n-3}, u_n\}$, $\{u, u_{n-2}, u_{n-1}\}$, $\{u, u_{n-2}, u_n\}$, $\{u, u_{n-1}, u_n\}$, $\{u_{n-3}, u_{n-2}, u_{n-1}\}$, $\{u_{n-3}, u_{n-2}, u_n\}$, $\{u_{n-2}, u_{n-1}, u_n\}$, $\{u_{n-3}, u_{n-1}, u_n\}$. Since u is of degree n – 2, u is adjacent every vertex of G except one of $u_{n-3}$, $u_{n-2}$, $u_{n-1}$, or $u_n$. Without loss of generality, let u be not adjacent with $u_{n-3}$. Then u is adjacent with $u_{n-2}$, $u_{n-1}$, $u_n$. Those sets containing u and $u_{n-2}$ or $u_{n-1}$ or $u_n$ are not independent. The left out four sets are $\{u_{n-3}, u_{n-2}, u_{n-1}\}$, $\{u_{n-3}, u_{n-2}, u_n\}$, $\{u_{n-2}, u_{n-1}, u_n\}$ and $\{u_{n-3}, u_{n-1}, u_n\}$. Consider $\{u_{n-3}, u_{n-2}, u_{n-1}\}$. u is of degree n – 2 and is strongly dominated by either $u_{n-2}$ or $u_{n-1}$. Therefore deg $u_{n-2} \geq n - 2$ or deg $u_{n-1} \geq n - 2$. Therefore $\{u_{n-3}, u_{n-2}, u_{n-1}\}$ is not an independent set. Therefore $\{u_{n-3}, u_{n-2}, u_{n-1}\}$ is not a strong efficient dominating set of G. By similar reasoning, the other three element sets are not strong efficient dominating sets of G. Therefore there exist no strong efficient dominating set of G of cardinality 3. Hence $\gamma_{se}(G) + \chi(G) \neq n + 1$. Therefore deg $u \leq n - 3$ for any u belongs to $V(K_{n-4})$. Thus deg $u_{n-3}$, deg $u_{n-2}$, deg $u_{n-1}$, deg $u_n$ and deg u for any u belongs to $V(K_{n-4}) \leq n - 3$.If $u_{n-3}(u_{n-2})$ is adjacent with $u_{n-1}$ and $u_n$ then deg $u_{n-3} = n - 2$ (deg $u_{n-2} = n - 2$ ) a contradiction. Similarly $u_{n-1}(u_n)$ can be adjacent with at most one of $u_{n-3}$, $u_{n-2}$. Any neighbour of $u_{n-2}$ is in $K_{n-4}$. Every vertex in $K_{n-4}$ is of degree n – 3 (Since deg u ≥ n – 3 for any u in V($K_{n-4}$). Also deg u ≤ n – 3 ). If u∈ $T_2$, then deg u = n – 3 > deg $u_{n-2}$ . Consider S = {u, $u_{n-2}$ , $u_n$}. Let v∈ (V – S) ∩ V($K_{n-4}$). Then v is strongly dominated by u. If deg $u_n$ = n – 3, then $u_n$ is adjacent with every vertex of $K_{n-4}$. v is strongly dominated by $u_n$, a contradiction. Thus deg $u_n$ ≤ n – 4. Therefore $u_n$ is not adjacent with some vertex of $K_{n-4}$. Let u be that vertex. Suppose this u is not adjacent with $u_{n-2}$. Then u is adjacent with $u_{n-3}$. The neighbours of $u_n$ of degree less than or equal to deg $u_n$ cannot be from $K_{n-4}$. $u_{n-2}$ and $u_{n-1}$ are not adjacent with $u_n$. Therefore only neighbours of $u_n$ other than those in $K_{n-4}$ is $u_{n-3}$. Let deg $u_{n-3}$ > deg $u_n$ . In this case S = {u, $u_{n-2}$ , $u_n$} is a strong efficient dominating set of G. If deg $u_{n-3}$ ≤ deg $u_n$, then $u_{n-3}$ is strongly dominated by u and $u_n$. Therefore S is not a strong dominating set of G.

**Case (iii)**: $u_{n-3}$ is adjacent with exactly one of $u_{n-1}$, $u_n$ and $u_{n-2}$ is adjacent with exactly one $u_{n-1}$, $u_n$. If $u_{n-3}$ and $u_{n-2}$ are both adjacent with $u_{n-1}$ (or both adjacent with $u_n$) then deg $u_{n-1}$ (or deg $u_n$) is n – 2, a contradiction.

**Sub case (i):** Let $u_{n-3}$ be adjacent with $u_{n-1}$, $u_{n-2}$ be not adjacent with $u_{n-1}$, $u_{n-2}$ be adjacent with $u_n$, $u_{n-3}$ be not adjacent with $u_n$. Since deg u ≤ n – 3, u is adjacent exactly with one $u_{n-3}$, $u_{n-2}$ and one of $u_{n-1}$, $u_n$. The possible three element sets from u, $u_{n-3}$, $u_{n-2}$, $u_{n-1}$ and $u_n$ are {u, $u_{n-3}$, $u_{n-2}$}, {u, $u_{n-3}$, $u_{n-1}$}, {u, $u_{n-3}$, $u_n$}, {u, $u_{n-2}$, $u_{n-1}$}, {u, $u_{n-2}$, $u_n$}, {u, $u_{n-1}$, $u_n$}, {$u_{n-3}$, $u_{n-2}$, $u_{n-1}$}, {$u_{n-3}$, $u_{n-2}$, $u_n$}, {$u_{n-2}$, $u_{n-1}$, $u_n$} and {$u_{n-3}$, $u_{n-1}$, $u_n$}. Nine of them are not independent. Consider {u, $u_{n-3}$, $u_n$}, deg u = n – 3 = deg $u_{n-3}$ = deg $u_{n-2}$ = deg $u_{n-1}$ = deg $u_n$. Therefore {$u_{n-3}$, u, $u_n$} is not a strong efficient dominating set of G. Therefore Case (iv) Sub case (i) does not arise.

**Illustration: 2.2** Consider the following graph G.

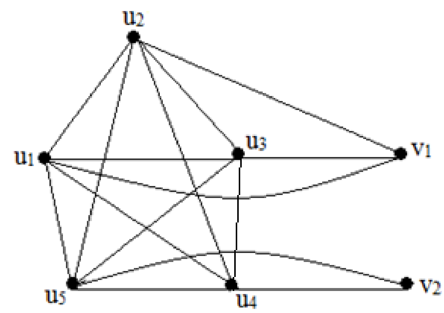


**Figure 1**

$\{u_i, v_1\}$, i = 4, 5 and $\{u_i, v_2\}$, i = 1, 2, 3 are $\gamma_{se}$-sets of G. Hence $\gamma_{se}(G) = 2$, $\chi(G) = 6$. Therefore $\gamma_{se}(G) + \chi(G) = 8 = 7 + 1$.

**Illustration: 2.3** Consider the following graph G.

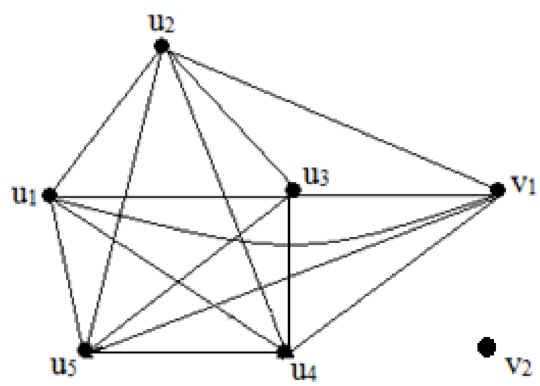


**Figure 2**

$\{u_i, v_2\}$, $1 \leq i \leq 5$ and $\{v_1, v_2\}$ are strong efficient dominating sets of G. Therefore $\gamma_{se}(G) = 2$.

$\chi(G) = 6$. Therefore $\gamma_{se}(G) + \chi(G) = 8 = 7 + 1$.

**Illustration: 2.4** Consider the following graph G.

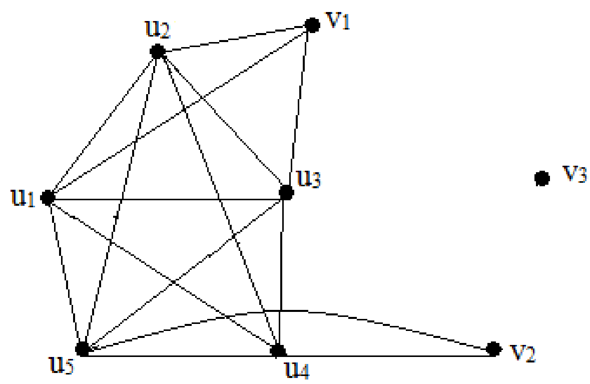


**Figure 3**

$\{u_i, v_2, v_3\}$ i = 1, 2, 3 and $\{u_i, v_1, v_3\}$ i = 4, 5 are $\gamma_{se}$ – sets of G. Therefore $\gamma_{se}(G) = 3$. $\chi(G) = 6$.

Therefore $\gamma_{se}(G) + \chi(G) = 9 = 8 + 1$.

**Illustration: 2.5** Consider the following graph G.

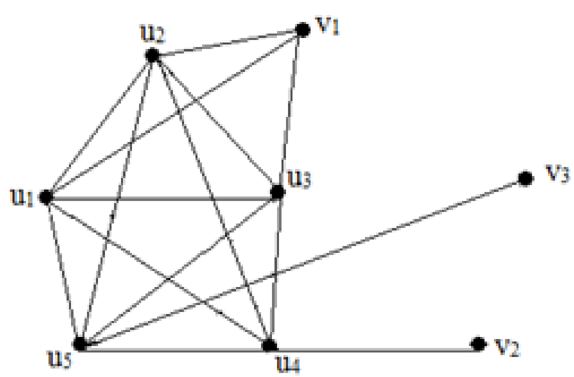


**Figure 4**

$\{u_i, v_2, v_3\}$ i = 1, 2, 3, $\{u_5, v_1, v_2\}$ and $\{u_4, v_1, v_3\}$ are $\gamma_{se}$ – sets of G. Hence $\gamma_{se}(G) = 3$, $\chi(G) = 6$.

Therefore $\gamma_{se}(G) + \chi(G) = 9 = 8 + 1$.

**Illustration: 2.6** Consider the following graph G.

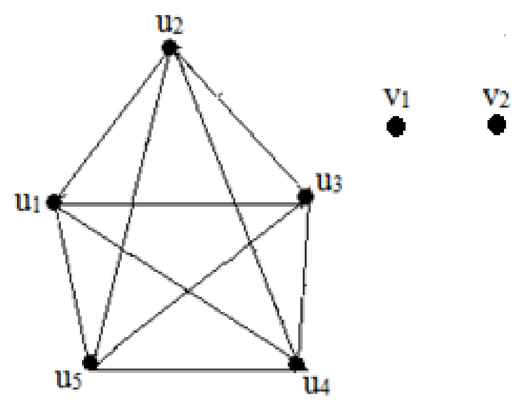


**Figure 5**

$\{u_i, v_1, v_2\}$, $1 \leq i \leq 5$ is a strong efficient dominating set of G. $\gamma_{se}(G) = 3$, $\chi(G) = 6$. Therefore $\gamma_{se}(G) + \chi(G) = 8 = 7 + 1$.

**Illustration: 2.7** Consider the following graph G.

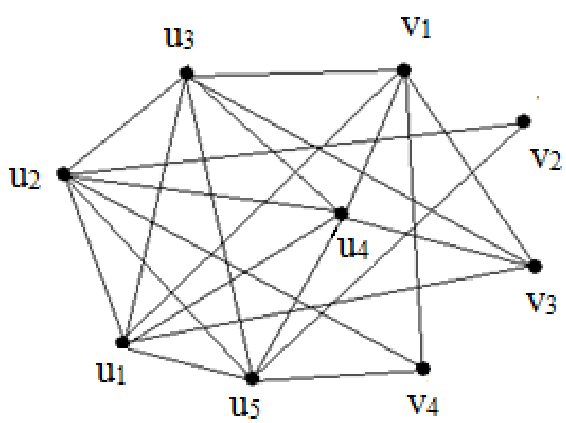


**Figure 6**

$\{u_1, v_2, v_4\}$, $\{u_2, v_1, v_3\}$, $\{u_3, v_2, v_4\}$, $\{u_4, v_2, v_4\}$, $\{u_5, v_1, v_3\}$ are $\gamma_{se}$ – sets of G. Therefore $\gamma_{se}(G) = 3$. $\chi(G) = 7$. Thus $\gamma_{se}(G) + \chi(G) = 10 = 9 + 1$.

**Illustration 2.8** Consider the following graph G.

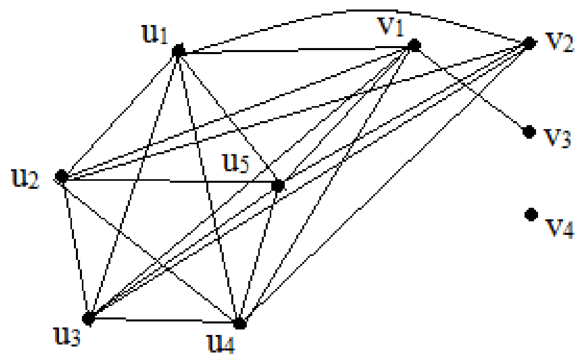


**Figure 7**

$\{u_i, v_3, v_4\}$ i = 1 to 5, $\{v_2, v_1, v_4\}$ are $\gamma_{se}$ – sets of G. Therefore $\gamma_{se}(G) = 3$. $\chi(G) = 7$.

Thus $\gamma_{se}(G) + \chi(G) = 10 = 9 + 1$.

**Illustration 2.9:** Consider the following graph G.

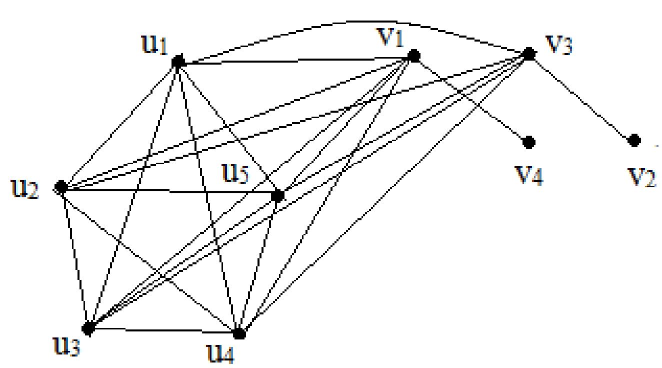


**Figure 8**

$\{u_i, v_2, v_4\}$ i = 1 to 5 are $\gamma_{se}$ – sets of G. Hence $\gamma_{se}(G) = 3$. $\chi(G) = 7$.

Thus $\gamma_{se}(G) + \chi(G) = 10 = 9 + 1$.

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