EFFICIENT STRONG VERTEX - EDGE DOMINATION SETS IN GRAPHS

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ABSTRACT

In this paper a new concept efficient strong vertex – edge domination set is introduced, efficient strong vertex –edge domination number of some well-known graphs, and bounds of the new parameter are identified. Changing and Unchanging properties of Efficient strong ve – domination number are discussed.

Key Words: Efficient domination, strong domination, vertex – edge domination, CER graph, UVR graph.

1. Introduction

1.1 Strong Domination:

Let *G = (V, E)* be a graph and $u, v \in V$. If $uv \in E$ *and* $\deg(u) \geq \deg(v)$ we say that u strongly dominates v and v is weakly dominates u. The minimum cardinality of a minimal strong dominating set is called **strong domination number** of a graph *G* and is denoted by $\gamma^{s}(G)$. .

1.2 Efficient Domination:

A dominating set D is said to be efficient domination set if for every vertex $u \in V$, $|N[u] \cap S|$= 1.Equantly a dominating set D is efficient if the distance between any two vertices in Sis at least 4, d (u,v) ≤ 4. The minimum cardinality of a minimal efficient dominating set is called **efficient domination number** of a graph *G* and is denoted by $\gamma_e(G)$.

1.3 Vertex – edge domination:

A set S ⊆V(G) is a **vertex-edge dominating set** if for all edges $e \in E(G)$, there exist a vertex v∈S such that v ve-dominates e. Otherwise for a graph *G = (V,E)* a vertex $u \in V(G)$

ve-dominates an edge $vw \in E(G)$ if (i) $u = v$ or $u = w$ (u is incident to vw), or (ii) uv or uw is an edge in G (u is incident to an edge that is adjacent to vw).

A subset S of V is said to be minimal vertex-edge dominating set if no proper subset of S is a vertex-edge dominating set of G.

The minimum cardinality of a minimal vertex-edge dominating set is called **vertex-edge domination number** of a graph G and is denoted by $\gamma_{ve}(G)$.

1.4 Efficient Strong vertex – edge domination:

A subset S of V is **efficient strong vertex-edge dominating set** if S is vertex-edge dominating set such that if for all edges $e \in E(G)$, there exist a vertex $v \in S$ such that v ve-dominates e.(i) If $uv \in E$ *and* $\deg(u) \geq \deg(v)$

(ii) for every vertex $u \in V$, $|N[u] \cap S| = 1$.

The minimum cardinality of a efficient strong vertex-edge dominating set is called **efficient strong vertex-edge domination number** of a graph G and is denoted by $\gamma^{s}_{eve}(G)$.

Theorem:1.5 For any Simple connected graph G $1 \leq \gamma^{s}_{eve}(G) \leq \frac{n}{4}$ for all n.

Proof: Let G be a graph with vertex set V (G) and edge set E (G).

To prove the lower bound, Let G be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges, then

$\gamma^{s}_{eve}(K_n) = 1$ since all the vertices adjacent.

To prove the upper bound, Let G be a simple connected graph with order n.

By the definition of efficient strong vertex edge domination a vertex $v \in S$ will ve- dominate an edge which is an incident edge of v and adjacent to incident edge and also for every $u,v \in S$

$d(u,v) \leq 4$, implies $\gamma^{s}_{eve}(G) \leq \frac{n}{4}$.

Thus $1 \leq \gamma^{s}_{eve}(G) \leq \frac{n}{4}$ for all n.

□

Proposition:1.6

The efficient strong vertex – edge domination number of path Pn is $\gamma^{s}{}_{eve} P_n = \left\lceil \frac{n}{4} \right\rceil$ Where n = 4,5,6…..

Proof:

In a path with n {x_1, x_2, x_3, … , x_n} vertices, n-1 edges {e_1, e_2, e_3, … , e_{n-1}}.

Let {x_1, x_2, x_3, … , x_n} $\in V(G)$, and vertex edge dominating vertices are

{x_4, x_8, x_{12}, … x_n } $\in$ S.

$|S| = \frac{n-4}{4} + 1$,

$= \frac{n-4+4}{4} \geq \frac{n}{4}$ for all n $\geq$ 4.------------------ (1)

Also for every $u,v \in S$ d(u,v) $\leq$ 4, implies $\gamma^{s}{}_{eve}(P_n) \leq \frac{n}{4}$.----------------- (2)

From (1) and (2) $\gamma^{s}{}_{eve} P_n = \left\lceil \frac{n}{4} \right\rceil$

Proposition:1.7

The efficient strong vertex – edge domination number of Cycle Cn is $\gamma^{s}{}_{eve} C_n = \frac{n}{4}$ Where n = 4,5,6…..

Proof: From the above proposition 1.6, we can prove that the efficient strong vertex – edge domination number of Cycle Cn is $\gamma^{s}{}_{eve} C_n = \frac{n}{4}$.

Proposition: 1.8

The efficient strong vertex – edge domination number of Complete graph Kn is $\gamma^{s}{}_{eve} K_n = 1$.

Proof: Let G be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. Since in a complete graph all the vertices are adjacent so that u $\in$ $V($Kn)will dominate all the edges and deg(u) $\leq$ deg(v).

Proposition: 1.9

The efficient strong vertex – edge domination number of Complete bi – partite graph $K_{n,m}$ is $\gamma^{s}{}_{eve} K_{n,m} = 1$.

Proof: From the above proposition 1.8, we can prove that the efficient strong vertex – edge domination number of Complete bi – partite graph $K_{n,m}$ is $\gamma^{s}{}_{eve} K_{n,m} = 1$.

Proposition: 1.10
The efficient strong vertex – edge domination number of star graph is $\gamma^s{}_{eve}\, K_{1,n-1} = 1$.
Proof: From the above proposition 1.8, we can prove that the efficient strong vertex – edge domination number of star graph is $\gamma^s{}_{eve}\, K_{1,n-1} = 1$.
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2. Changing and Unchanging Properties

2.1 Changing Edge Removal Graph and Unchanging Vertex Removal Graph

A graph G is said to be a CER – graph if $\gamma^s{}_{eve}\,(G-e) \neq \gamma^s{}_{eve}\,(G)$ *for each* $e \in E$.

A graph G is said to be a UVR – graph if $\gamma^s{}_{eve}\,(G-u) = \gamma^s{}_{eve}\,(G)$ *for each* $u \in V$.

The vertex set and edge set are partitioned into three sets depending on the effect of removal of vertex or edge.

V^0 = {u∈V : $\gamma^s{}_{eve}\,(G-u) = \gamma^s{}_{eve}\,(G)$ }

V^+ = {u∈V : $\gamma^s{}_{eve}\,(G-u) > \gamma^s{}_{eve}\,(G)$ }

V^- = {u∈V : $\gamma^s{}_{eve}\,(G-u) < \gamma^s{}_{eve}\,(G)$ }

E^0 = {e∈E: $\gamma^s{}_{eve}\,(G-e) = \gamma^s{}_{eve}\,(G)$ }

E^+ = {e∈E : $\gamma^s{}_{eve}\,(G-e) > \gamma^s{}_{eve}\,(G)$ }

E^- = {e∈E : $\gamma^s{}_{eve}\,(G-e) < \gamma^s{}_{eve}\,(G)$ }

Theorem: 2.2

Let G be a CER – graph then for each $\gamma^s{}_{eve}$ - set S of G, each edge e of G joins S and V-S.

Proof: Let G be CER – graph, and S be a $\gamma^s{}_{eve}$ set.

Suppose there is an edge e which lies in <V-S>, where S is an efficient strong vertex edge dominating set of G. Let S_1be a $\gamma^s{}_{eve}$ set of G-e such that $\gamma^s{}_{eve}\,(G-e) = S_1$ *and* $\gamma^s{}_{eve}\,(G) = S$.
Since S is an vertex edge dominating set, each vertex will dominate incident edge and adjacent to incident edge, also for every vertex $u \in V$, $|N[u] \cap S|$ = 1 which implies, $\gamma^s{}_{eve}\,(G-e) < \gamma^s{}_{eve}\,(G)$,

Theorem:2.3 Cycle C_n preserves Unchanging vertex removal property if n is even.

Proof: Let C_n be a cycle with n vertices and n edges, where n is odd.

Let u $\in$ V(C_n), $\gamma^s{}_{eve}(C_n - u) = \gamma^s{}_{eve}(P_{n-1}) = \gamma^s{}_{eve}(C_n)$. There fore $C_n \notin$ UVR, which is a contradiction, hence n is even. We know that $\gamma^s{}_{eve}(P_{n-1}) = \gamma^s{}_{eve}(C_n) = \frac{n}{4}$ for all n is 4, 6,8,…. Therefore $\gamma^s{}_{eve}(C_n - u) = \gamma^s{}_{eve}(C_n)$.

Observation: 2.4 Any UVR graph cannot have isolates.

Theorem: 2.5 If T is a tree of diameter 2, then T $\in$ UVR if and only if T = $K_{1,2}$.

Proof: If the diameter of T is 2 then T = $K_{1,n}$, n $\geq$2 and $\gamma^s{}_{eve} K_{1,n} = 1$. Let u be the central vertex of $K_{1,n}$. T $\in$ UVR if and only if $\gamma^s{}_{eve}(T - v) = 1$ for each vertex v. Now for any v $\neq$ u $\gamma^s{}_{eve}(T - v) = 1$ and $\gamma^s{}_{eve}(T - u) = \gamma^s{}_{eve}(nK_1) = n$. Therefore $\gamma^s{}_{eve}(T - u) = \gamma^s{}_{eve}(nK_1) = n =$ 1. Thus T = $K_{1,2}$.

Conclusion:

In this paper a new concept efficient strong vertex – edge domination set is introduced and bounds were discussed. Introduction concept of Changing , Unchanging properties were discussed and planning to identify more ideas in this property as a future work.

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