BALANCED DOMINATION NUMBER OF SPECIAL GRAPHS

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ABSTRACT

Let G= (V, E) be a graph. A Subset D of V is called a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The Domination number γ (G) of G is the cardinality of the minimum dominating set of G.

Let G = (V, E) be a graph and let f be a function that assigns to each vertex of V to a set of values from the set {1,2,.......k} that is, f:V(G) → {1,2,.....k} such that for each u,v ∈ V(G), f(u) ≠ f(v), if u is adjacent to v in G. Then the dominating set D ⊆ V (G) is called a balanced dominating set if $\sum_{u \epsilon D} f(u) = \sum_{v \epsilon V-D} f(v)$. In this paper, we investigate the balanced domination number for some special graphs.

Keywords: balanced, domination, special graph

Mathematics subject classification: 05C69

INTRODUCTION

Let G = (V, E) be a graph and let f be a function that assigns to each vertex of V to a set of values from the set {1,2,.......k} that is, f:V(G) → {1,2,.....k} such that for each u,v ϵ V(G), f(u)≠f(v), if u is adjacent to v in G. Then the set D ⊆V (G) is called a balanced dominating set if $\sum_{u \epsilon D} f(u) = \sum_{v \epsilon V-D} f(v)$.

The balanced domination number $\gamma_{bd}(G)$ is the minimum cardinality of the balanced dominating set.

1. The set D ⊆ V (G) is called strong balanced dominating set if $\sum_{u \epsilon D} f(u) \geq \sum_{v \epsilon V-D} f(v)$.Also the set D ⊆ V (G) is called weak balanced dominating set if $\sum_{u \epsilon D} f(u) \leq \sum_{v \epsilon V-D} f(v)$

2. The sum of the values assigned to each vertex of G is called the total value of G. that is, Total value= f (V) = $\sum_{v \epsilon V(G)} f(v)$.

Theorem 1

Let G be a graph with n vertices. Then G has a balanced dominating set iff f (V) = $\sum_{v \epsilon V(G)} f(v)$ is even.

Proved in [6].

Theorem 2

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Let G be a graph with n vertices. Then G has no balanced dominating set iff f (V) = $\sum_{v \in V(G)} f(v)$ is odd.

Proved in [6].

2. Special graphs

1) The **Bidiakis cube** is a 3-regular graph with 12 vertices and 18 edges as shown in Figure 1.

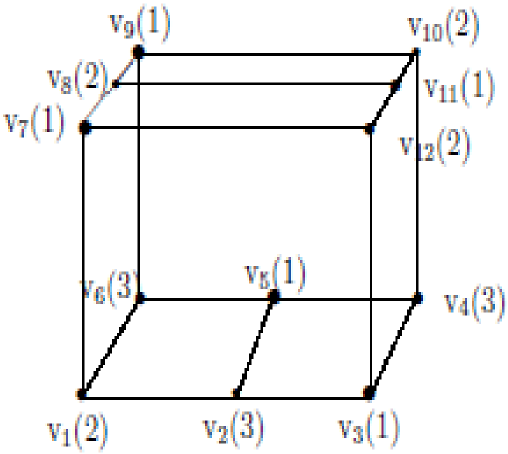

Figure 1: Bidiakis cube

For the Bidiakis cube graph, Υ_{bd}=5

Here f (V) = 22 and D= {v_2, v_6, v_8, v_{10}, v_{11}} is a balanced dominating set.

$\sum_{v \in D} f(v)$= 3+3+2+2+1= 11

Therefore, Υ_{bd}=5

2) The **Franklin graph** is a 3-regular graph with 12 vertices and 18 edges as shown in Figure 2.

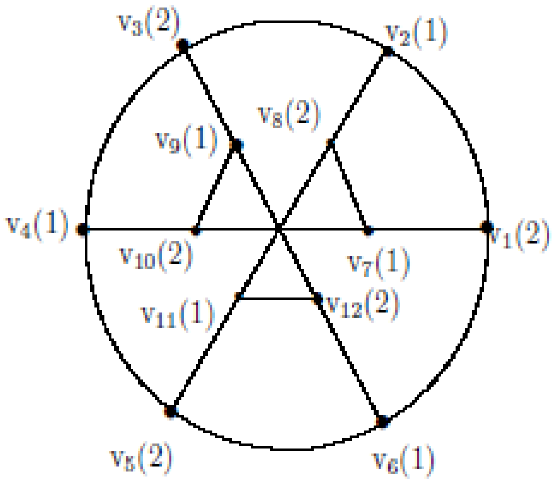

Figure 2: Franklin graph

For the Franklin graph, Y_{bd}=5

Here f (V) = 18 and D= {v_2, v_6, v_8, v_{10}, v_{12}} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 2+2+2+2+1= 11

Therefore, Y_{bd}=5

3) The **Frucht graph** is a 3-regular graph with 12 vertices, 18 edges, and no nontrivial symmetries as shown in Figure 3.

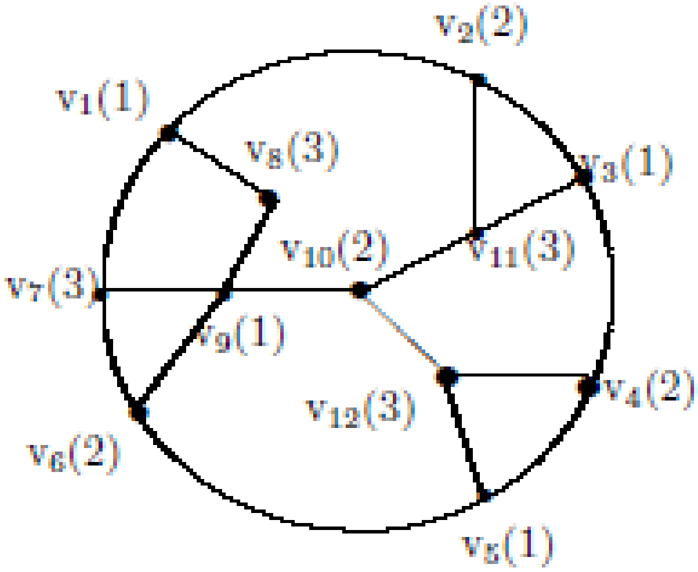

Figure 3: Frucht graph

For the Frucht graph, Υ_{bd}=4

Here f (V) = 24 and D= {v_7, v_8, v_{11}, v_{12}} is a balanced dominating set.

$\sum_{v \in D} f(v)$= 3+3+3+3= 12

Therefore, Υ_{bd}=4

4) The **Wagner graph** is a 3-regular graph with 8 vertices and 12 edges as shown in Figure 4, named after Klaus Wagner. It is the 8-vertex Mobius ladder graph. Mobius ladder is a cubic circular graph with an even number

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'n' vertices, formed from an n-cycle by adding edges connecting opposite pairs of vertices in the cycle.

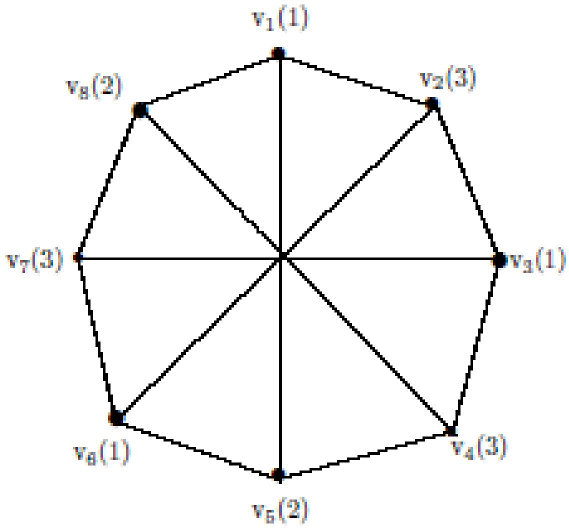

Figure 4 : Wagner graph

For the Wagner graph, Y_{bd}=3

Here f (V) = 16 and D= {v_2, v_5, v_7} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 3+3+2= 8

Therefore, Y_{bd}=3.

5) The **Herschel graph** is a bipartite undirected graph with 11 vertices and 18 edges as shown in Figure 5, the smallest non Hamiltonian polyhedral graph. It is named after British astronomer Alexander Stewart Herschel.

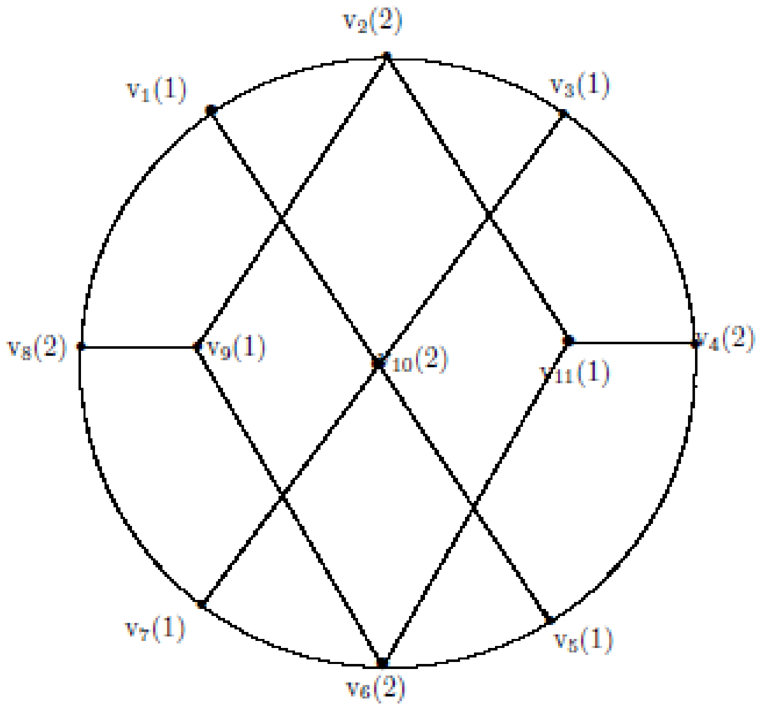

Figure 5: Herschel graph

For the Herschel graph, Υ_{bd}=5

Here f (V) = 16 and D= {v_4, v_8, v_9, v_{10}, v_{11}} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 2+2+2+1+1= 8

Therefore, Υ_{bd}=5

6) Moser spindle (also called the Moser's spindle or Moser graph) is an undirected graph, named after mathematicians Leo Moser and his brother William with seven vertices and eleven edges as shown in Figure 7.

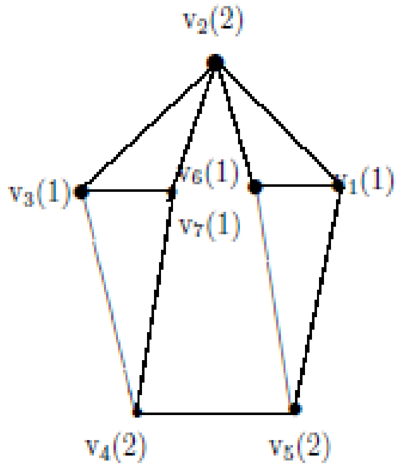

Figure 6 : Moser spindle

For the Moser spindle graph, Υ_{bd}=3

Here f (V) = 10 and D= {v_2, v_3, v_5} is a balanced dominating set.

$\sum_{v \epsilon D} f(v)$= 2+2+1= 5

Therefore, Υ_{bd}=3

7) The **Goldner-Harary graph** is a simple undirected graph with 11 vertices and 27 edges as shown in Figure 7. It is named after A. Goldner and Frank Harary, who proved in 1975 that it was the smallest non Hamiltonian maximal planar graph.

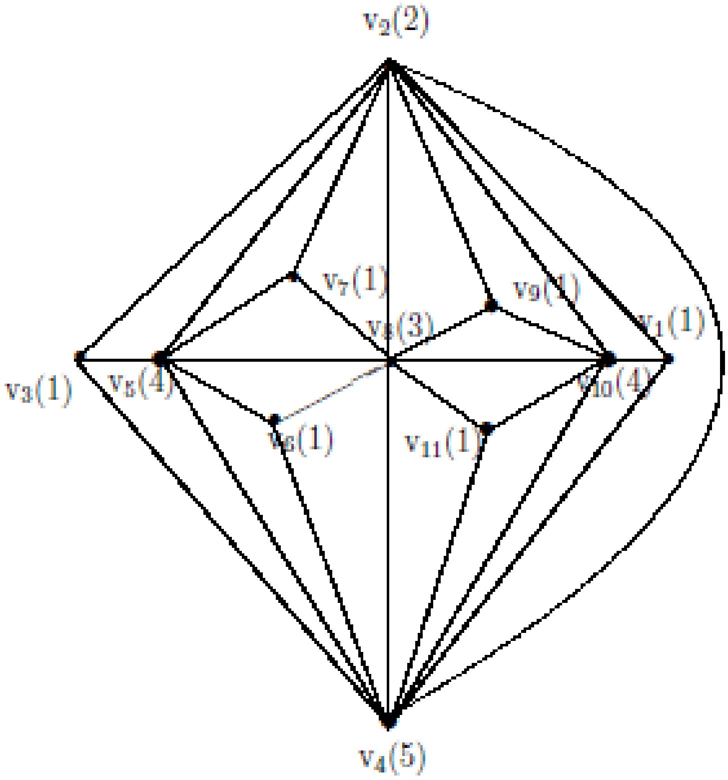

Figure 7 : Goldner-Harary graph

For the Goldner-Harary graph, $\Upsilon_{bd}=3$

Here f(V)= 24 and D= $\{v_4, v_8, v_{10}\}$ is a balanced dominating set.

$\sum_{v \in D} f(v)$= 5+4+3= 12

Therefore, $\Upsilon_{bd}=3$.

8) The **Grotzch graph** is a triangle-free graph with 11 vertices, 20 edges, chromatic number 4 and crossing number 5 as shown in Figure 8. It is named after German mathematician Herbert Grotzsch.

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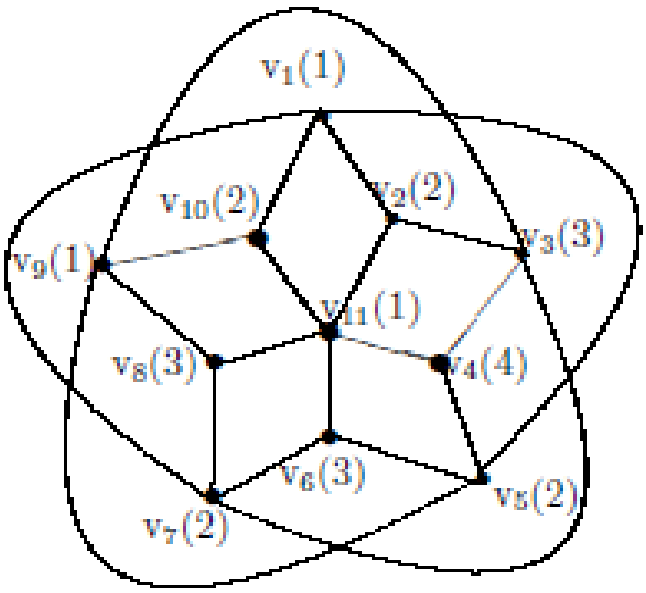

Figure 8 : Grotzch graph

For the Grotzch graph, Y_{bd}=5

Here f(V)= 24 and D= { v_3,v_4, v_5, v_{10}, v_{11}} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 4+3+2+2+1= 12

Therefore, Y_{bd}=5.

9) The **Hoffman graph** is a 4-regular graph with 16 vertices and 32 edges as shown in Figure 9 discovered by Alan Hoffman.

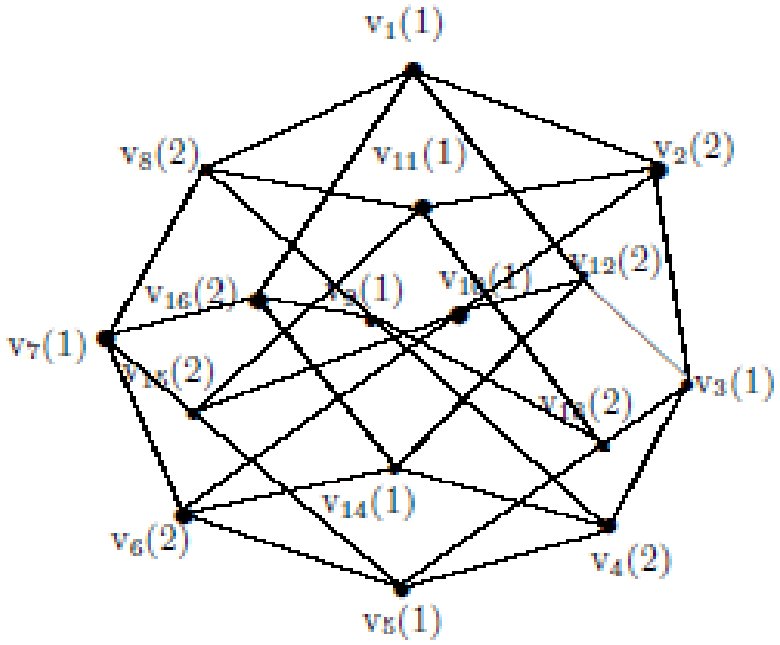

Figure 9 : Hoffman graph

For the Hoffman graph, Υ_{bd}=7

Here f(V)= 24 and D= { $v_4, v_6, v_8, v_{11}, v_{12}, v_{14}, v_{15}$} is a balanced dominating set.

$\sum_{v \in D} f(v)$= 2+2+2+2+2+1+1= 12

Therefore, Υ_{bd}=7.

10) The **Mobius-Kantor graph** is a symmetric bipartite cubic graph with 16 vertices and 24 edges as shown in Figure 10, named after August Ferdinand Mobius and Seligmann Kantor.

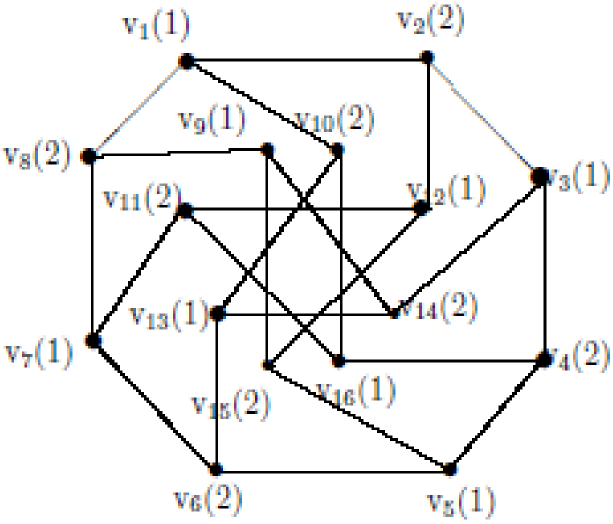

Figure 10 : Mobius-Kantor graph

For the Mobius-Kantor graph, Υ_{bd}=7

Here f(V)= 24 and D= { v_2,v_3, v_8, v_{10}, v_{11}, v_{13}, v_{15}} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 2+2+2+2+2+1+1= 12

Therefore, Υ_{bd}=7.

11) The **Truncated Tetrahedron** is an Archimedean solid. It has 4 regular faces, 4 regular triangular faces, 12 vertices and 18 edges as shown in Figure 11. Archimedean solid means one of 13 possible solids whose faces are all regular polygons whose polyhedral angles are all equal.

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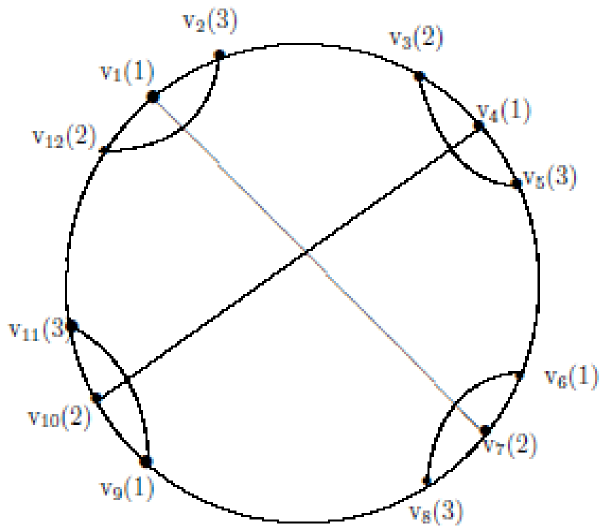

Figure 11 : Truncated Tetrahedron

For the Truncated Tetrahedron graph, Υ_{bd}=4

Here f(V)= 24 and D= { v_2,v_5, v_8, v_{11}} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 3+3+3+3= 12

Therefore, Υ_{bd}=4.

12) The **Desargues graph** is a distance-transitive graph with 20 vertices and 30 edges as shown in Figure 12. It is named after Gerard Desargues.

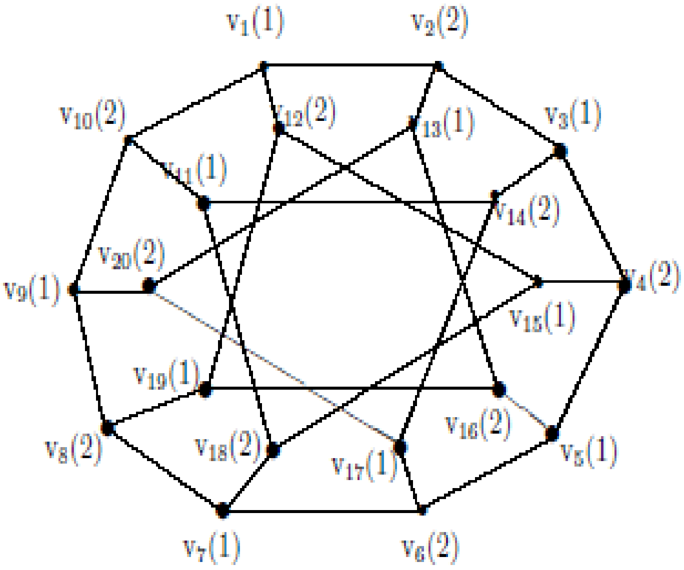

Figure 12: Desargues graph

For the Desargues graph, Υ_{bd}=8

Here f(V)= 30 and D= { v_2,v_4, v_6, v_8, v_{11}, v_{12}, v_{16}, v_{20}} is a balanced dominating set.

$\sum_{v\epsilon D} f(v)$= 2+2+2+2+2+2+2+1= 15

Therefore, Υ_{bd}=8.

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